Stat 411/511

One sample (paired) t-Test

Jan 25 2012

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Reminder

TODAY: Help session (after lecture 2.30pm to 4pm CORD 2113)

TOMORROW: 2-3.30pm Valley Library 6420 (this week only)

I don't prepare anything. You bring questions or the lab to work on. I answer questions and help if you get stuck.

The Schizophrenia case study

15 identical twins, one with schizophrenia.

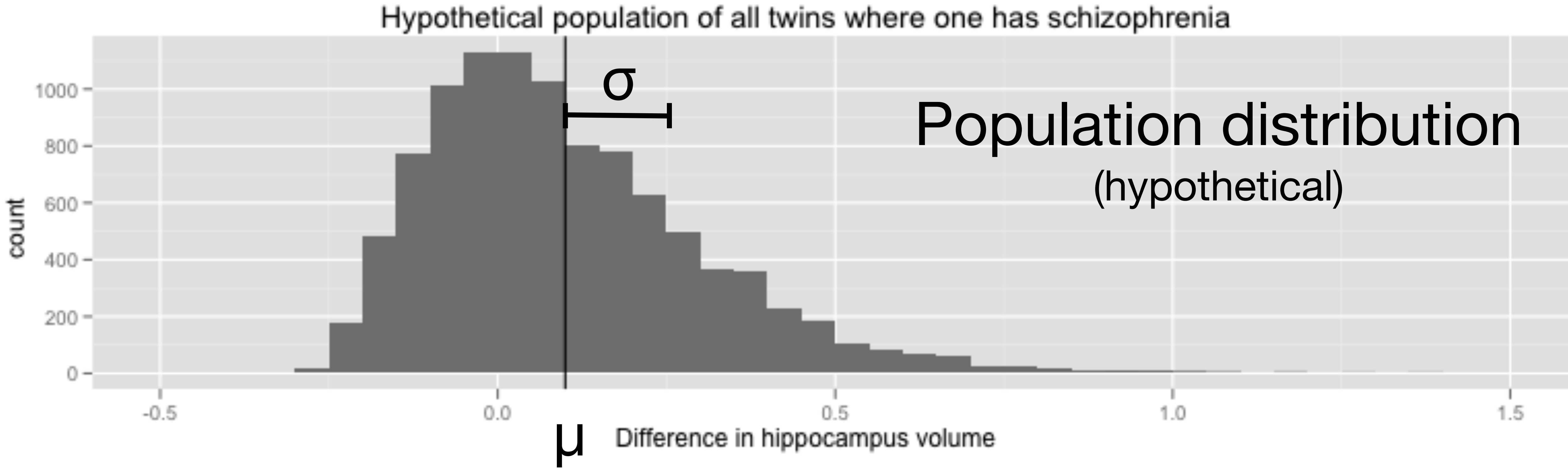
Measurements on left hippocampus volume.

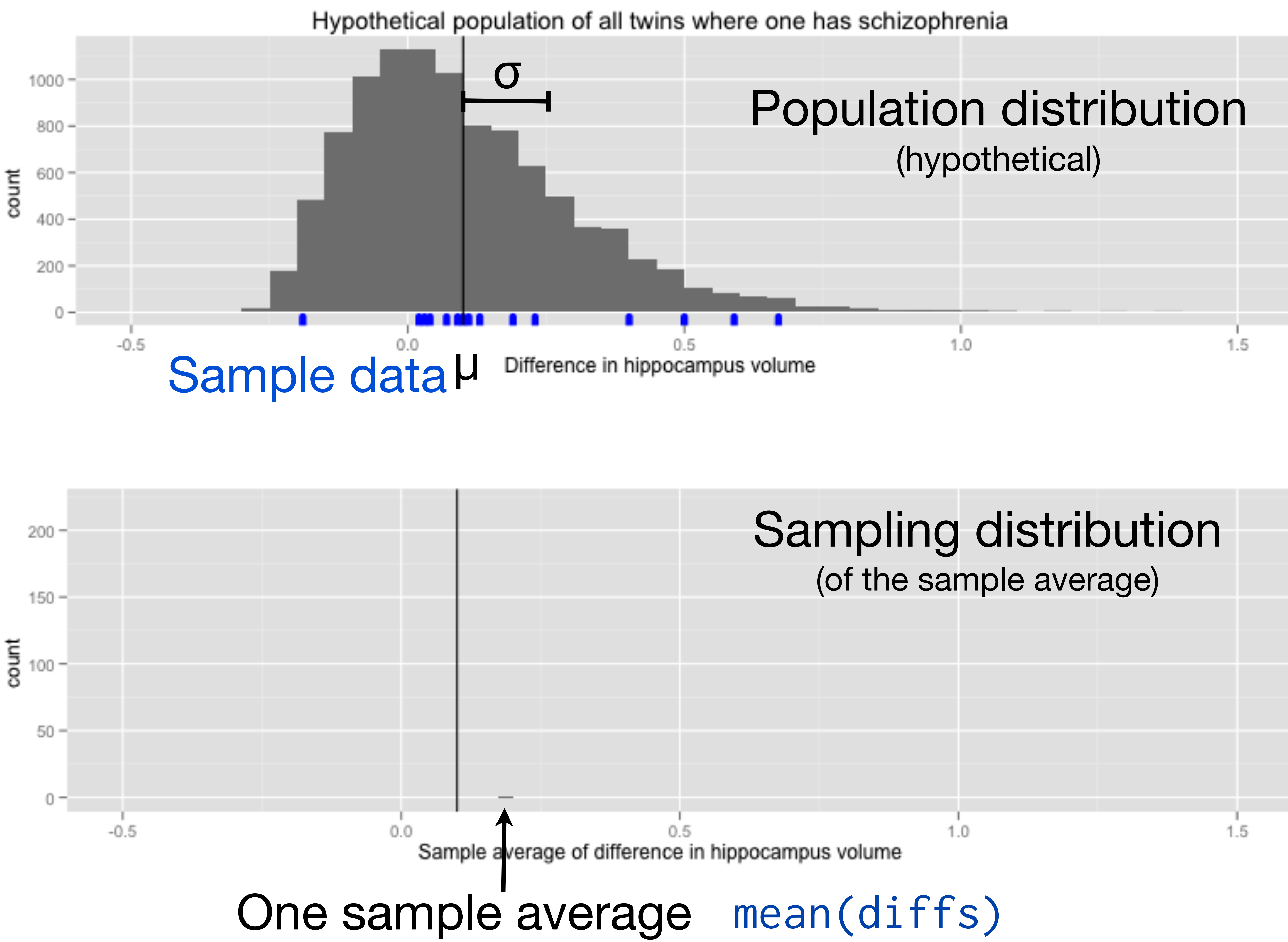
Naturally **paired**. Look at the **differences** between twins.

```
diffs <- case0202$Unaffect - case0202$Affected
```

Two questions

- Is the mean difference in volume, μ, different from zero?
- What's a likely range for μ?


Hypothetical population of all twins where one has schizophrenia
σ
Population distribution
(hypothetical)
count
1000
800
600
400
200
0
-0.5
0.0
0.5
1.0
1.5
μ
Difference in hippocampus volume



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count
1000
800
600
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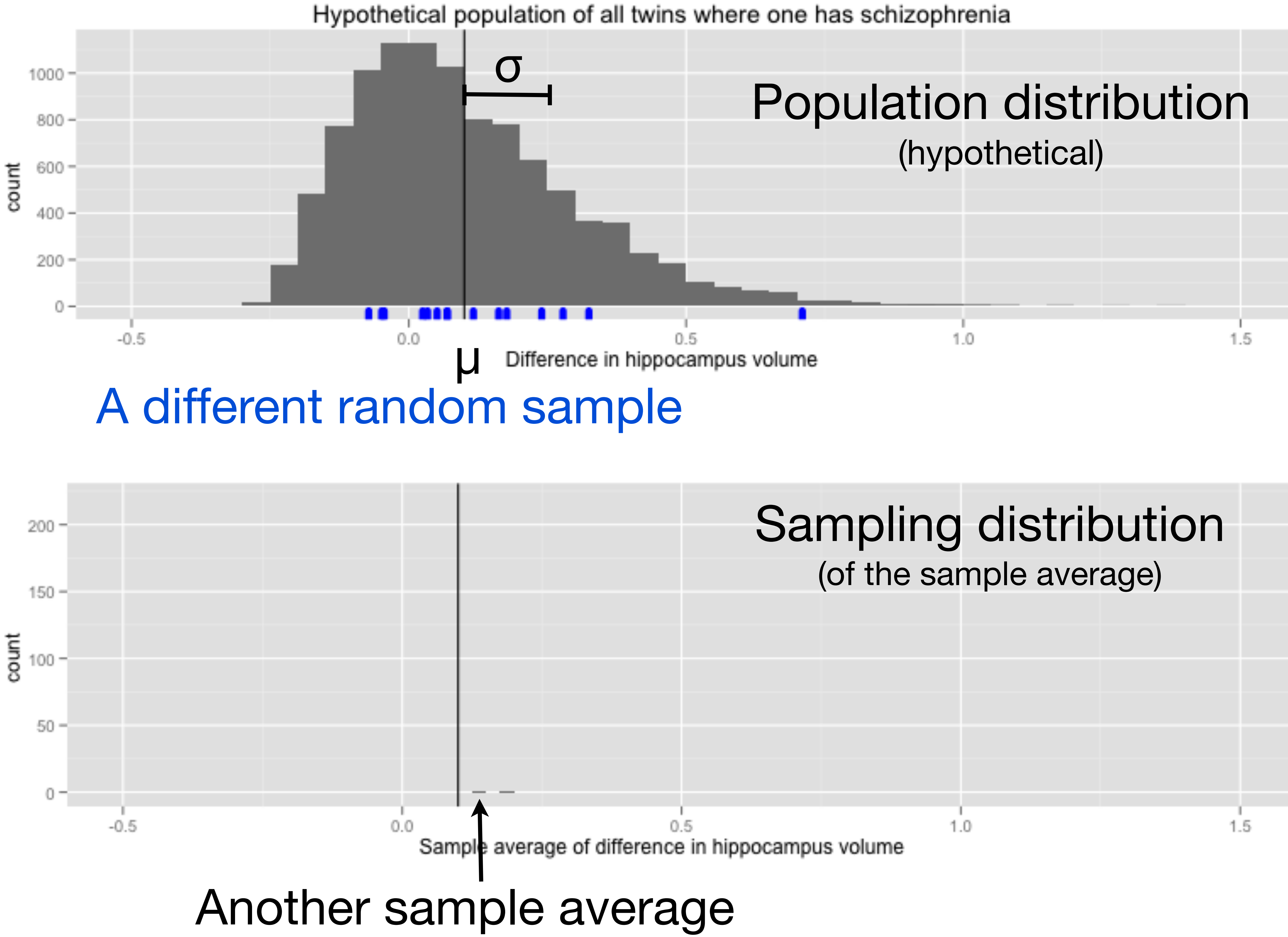
0.0
0.5
1.0
1.5
Sample data
μ
Difference in hippocampus volume
Sampling distribution
(of the sample average)
count
200
150
100
50
0
-0.5
0.0
0.5
1.0
1.5
Sample average of difference in hippocampus volume
One sample average
mean(diffs)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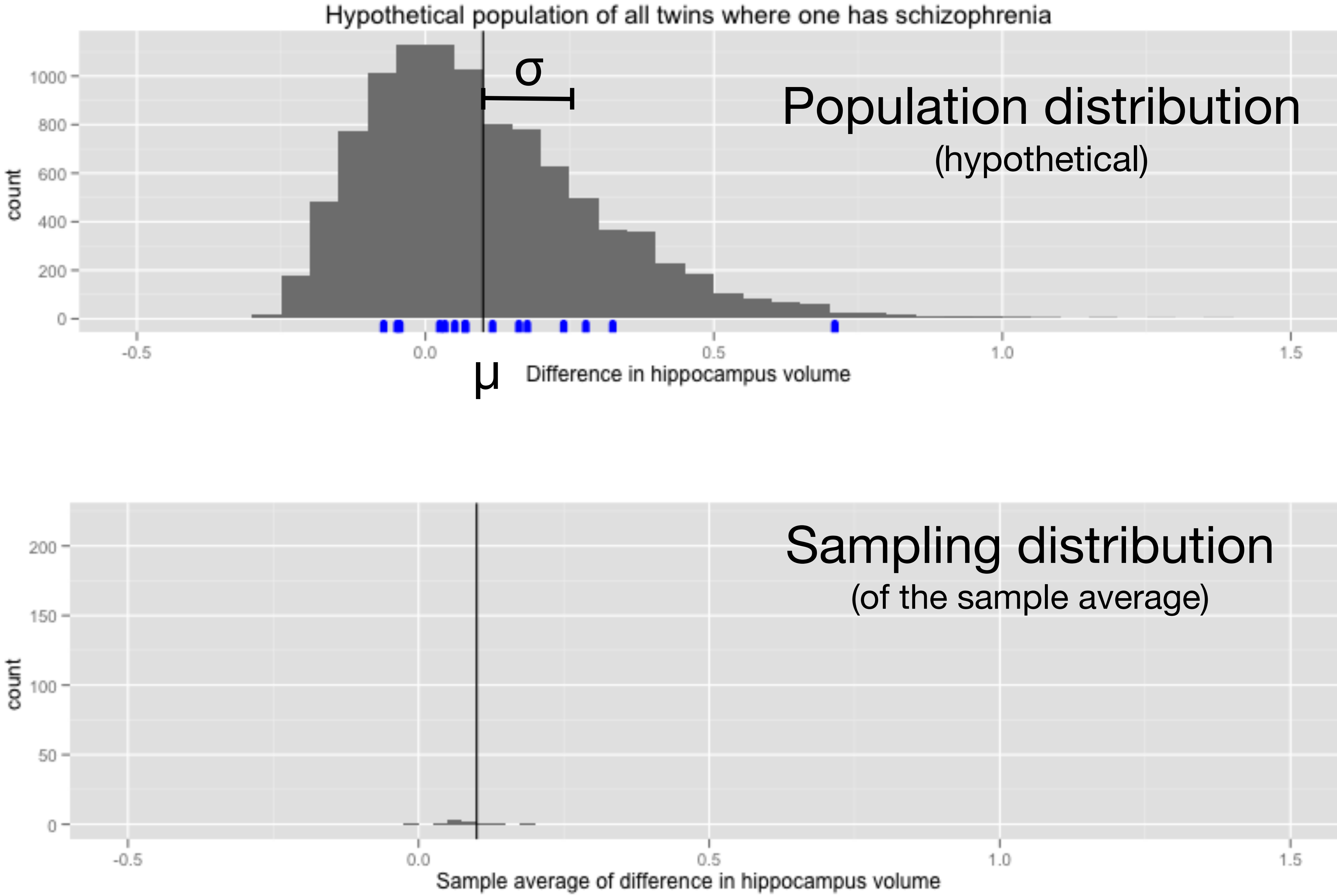
0.0
0.5
1.0
1.5
μ
Difference in hippocampus volume
A different random sample
Sampling distribution
(of the sample average)
count
200
150
100
50
0
-0.5
0.0
0.5
1.0
1.5
Sample average of difference in hippocampus volume
Another sample average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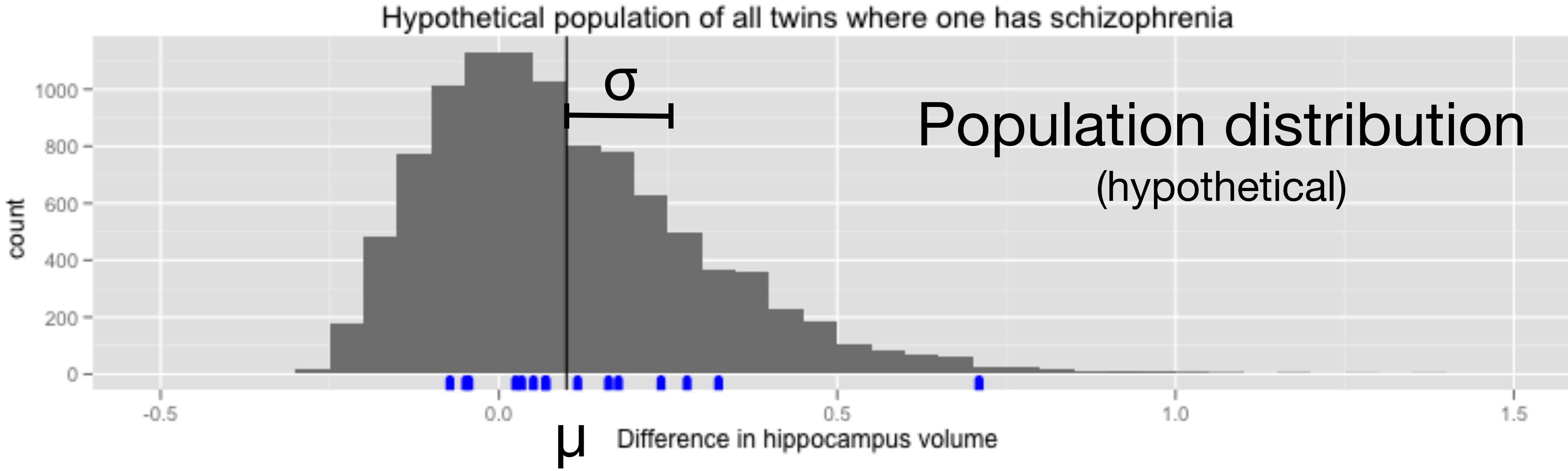
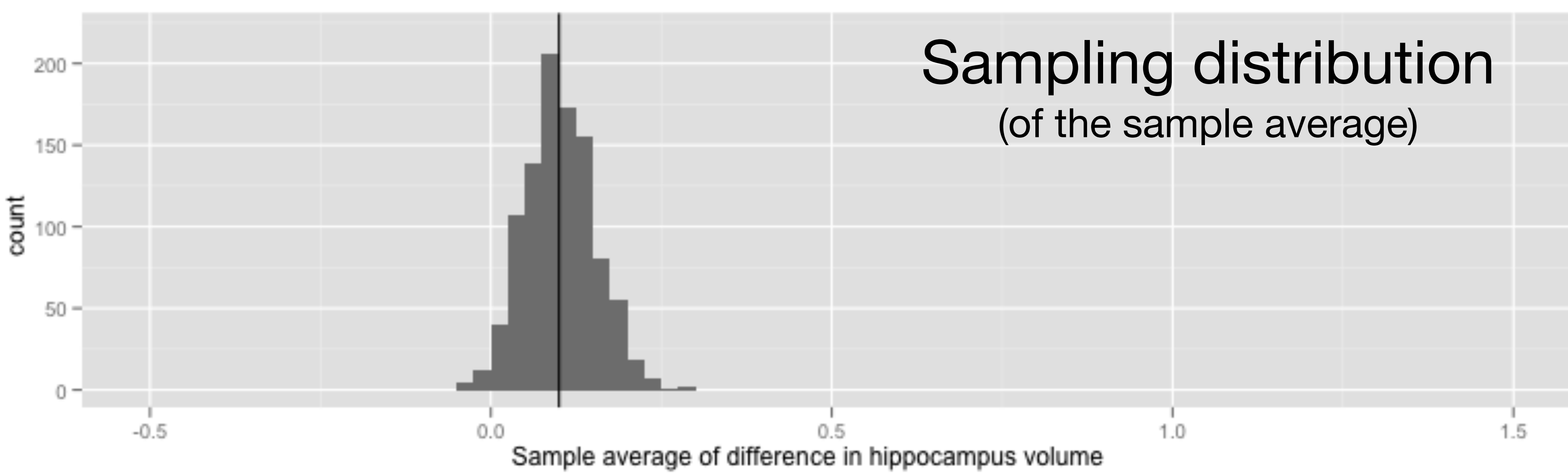
Ten more





A thousand more

Your turn

Compare the center, spread and shape of the population distribution and the sampling distribution.

Fact #1

If a population has mean μ and standard deviation σ, then the average of a sample of n subjects, $\overline{Y}$, has mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$

Fact #2

The shape of the sampling distribution of $\overline{Y}$ is more nearly normal that the population distribution

(Central Limit Theorem)

Standard Error (some terminology)

The **standard deviation** of $\overline{Y}$ is $\frac{\sigma}{\sqrt{n}}$

But we don't know σ (it's a parameter)

We can **estimate** σ with s (the sample standard deviation).

We call the estimate of the standard deviation of $\overline{Y}$ the **standard error** of $\overline{Y}$

$$\mathrm{SE}_{\overline{Y}} = \frac{s}{\sqrt{n}} = \frac{0.238}{\sqrt{15}} = 0.0615$$

```
sd(diffs)/sqrt(length(diffs))
```

Degrees of freedom

Every standard error has a accompanying degrees of freedom.

This summarizes how much (independent) data went into the variability calculation.

For the **standard error of the sample average** in a **single sample** the degrees of freedom is **one less than the number of subjects**.

d.f. = n -1

d.f. = 15 -1 = 14 in the schizophrenia case

The *t*-ratio

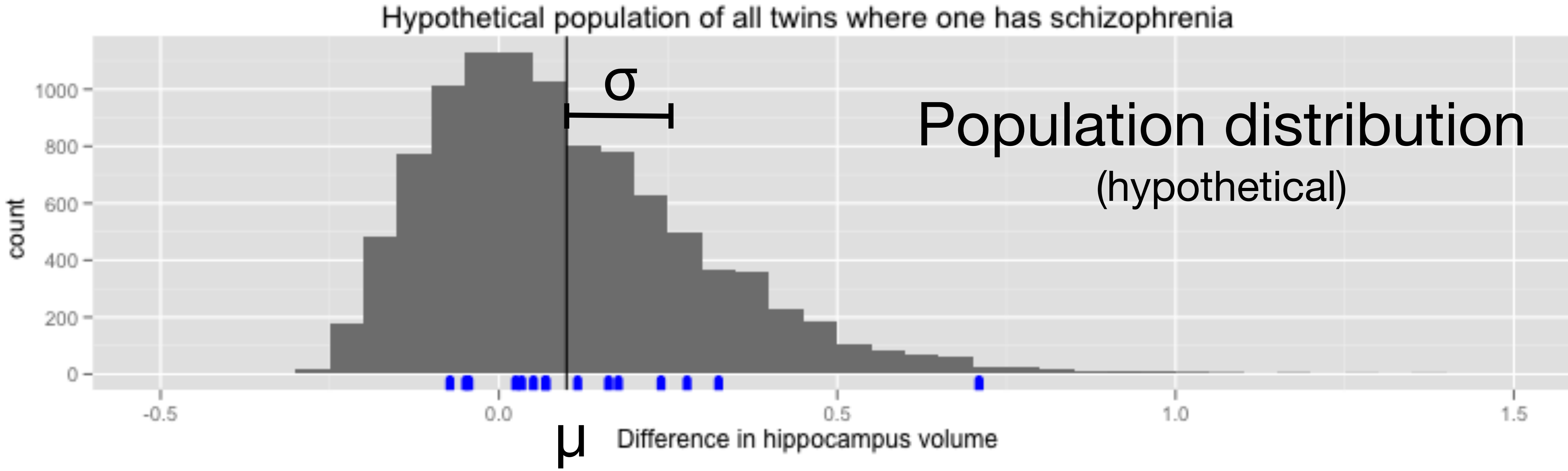
$$t\text{-ratio} = \frac{\overline{Y} - \mu}{\text{SE}} = \frac{\overline{Y} - \mu}{\frac{s}{\sqrt{n}}}$$

A measure of **how far** the **sample average** is from the **population mean** in **relative** to the **variability** we expect in the **sample average**

Fact #3

If $\overline{Y}$ is the average of a sample of **n** subjects from a **normally distributed** population, then it's t-ratio is described by a

Student't *t*-distribution with **n-1** degrees of freedom.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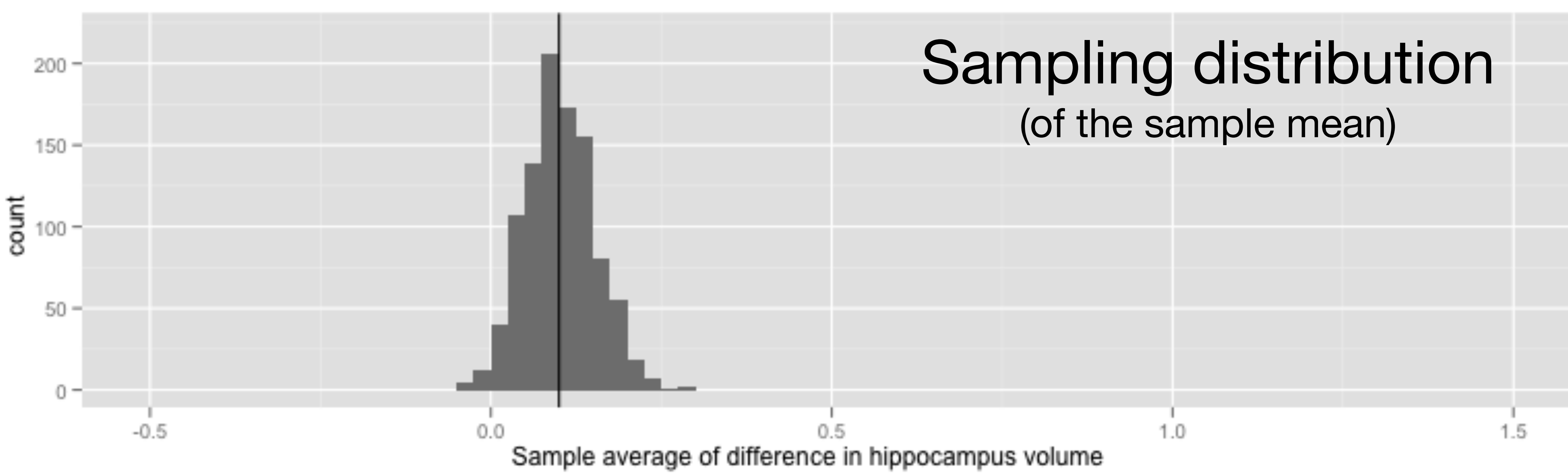
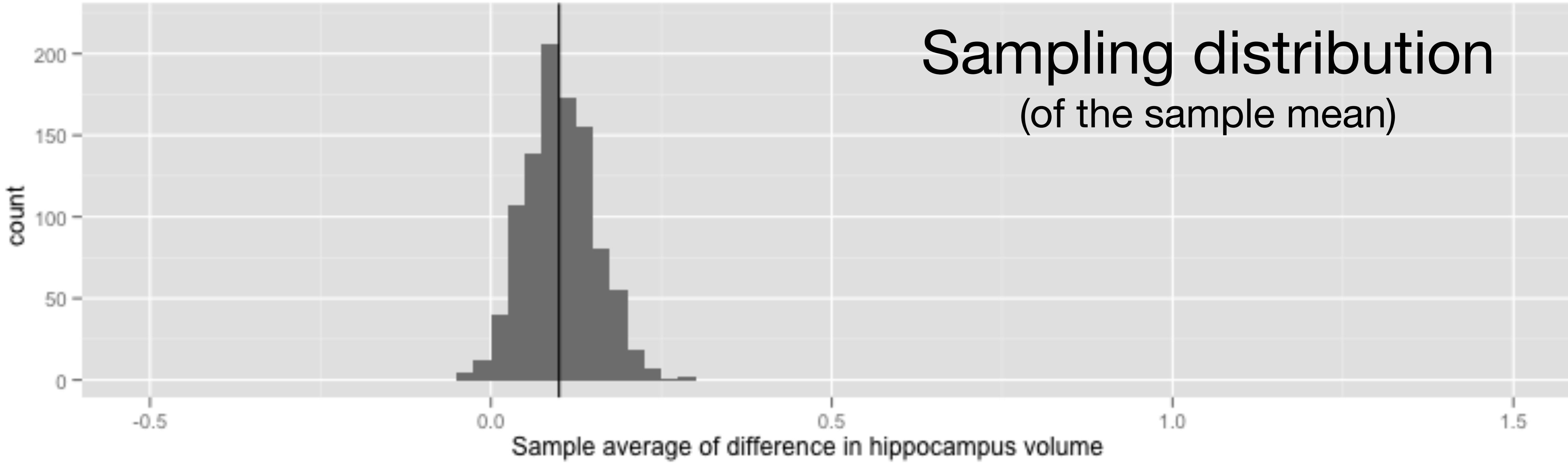

Sampling distribution
(of the sample mean)
count
200
150
100
50
0
-0.5
0.0
0.5
1.0
1.5
Sample average of difference in hippocampus volume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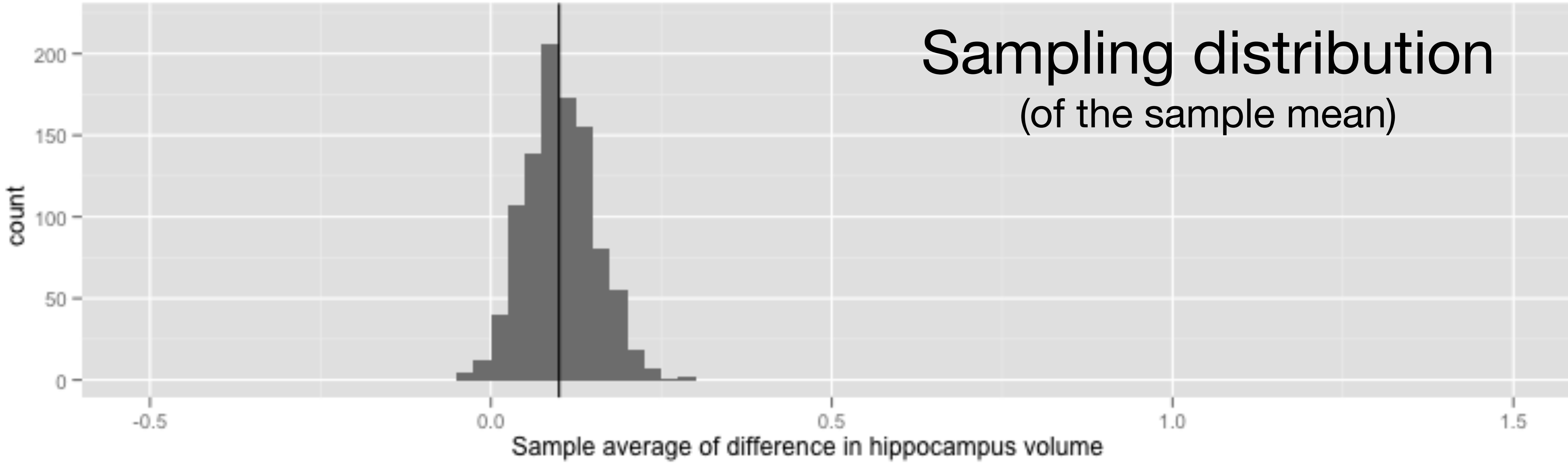
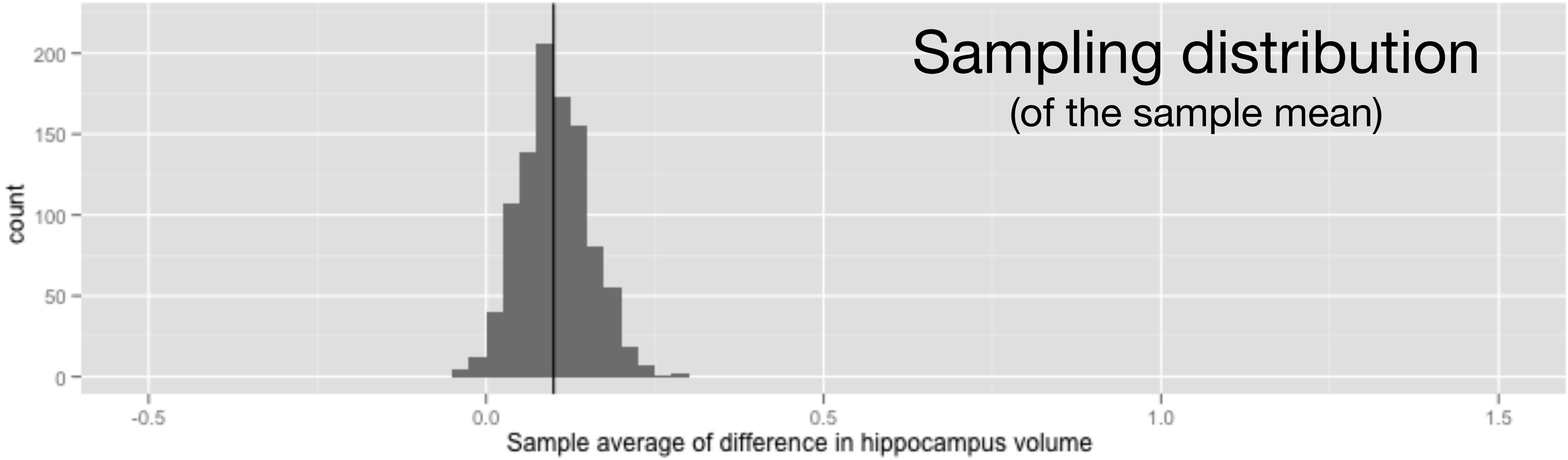


Sampling distribution

(of the sample mean)

t-ratio

(subtract mean and
divide by standard error)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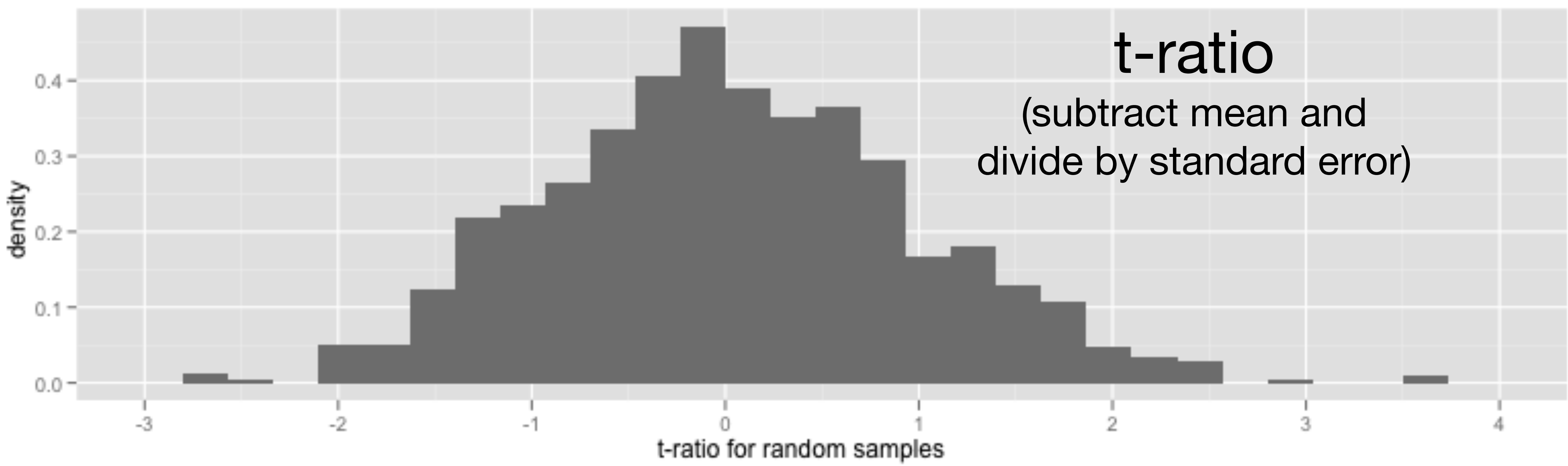
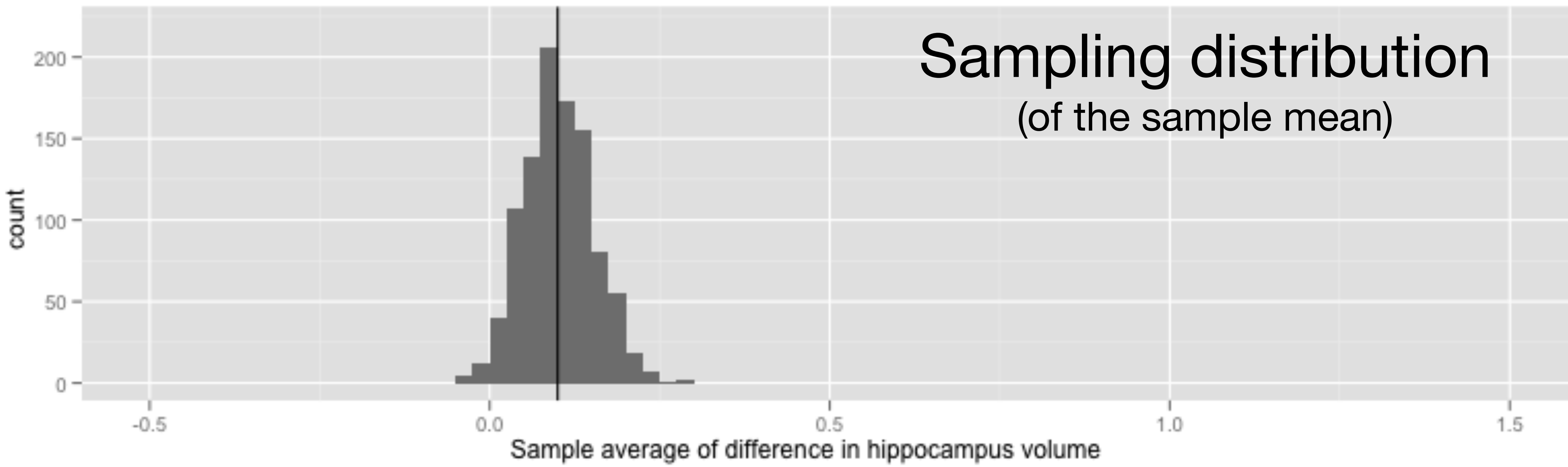

t-ratio
(subtract mean and
divide by standard error)
density
0.4
0.3
0.2
0.1
0.0
-3
-2
-1
0
1
2
3
4
t-ratio for random samples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200
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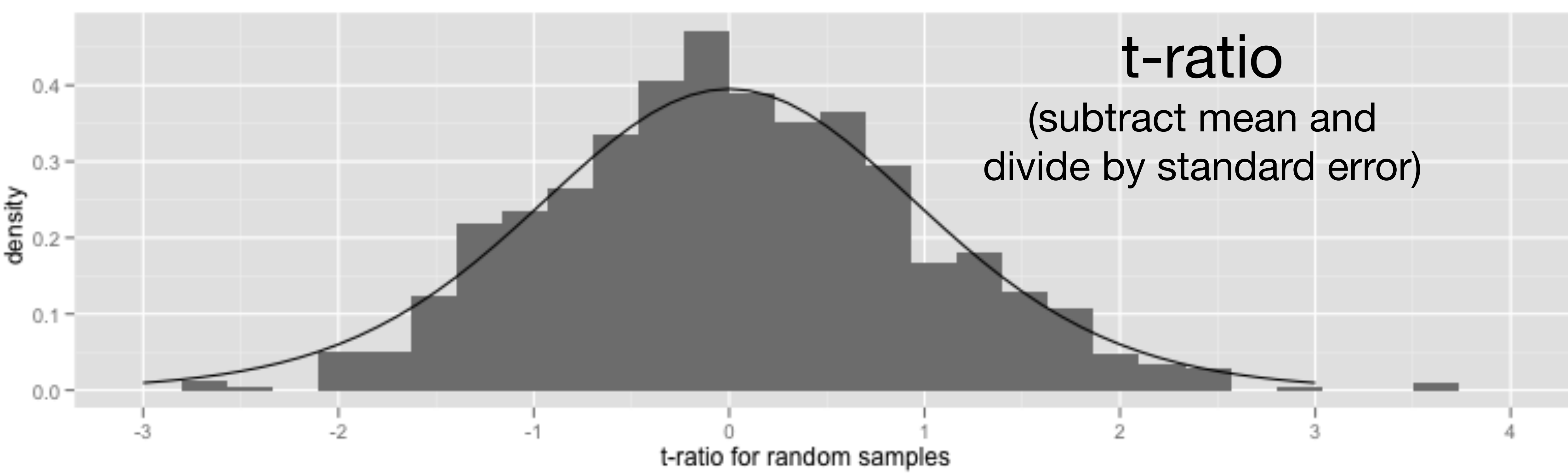
Sample average of difference in hippocampus volume


t-ratio
(subtract mean and
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0.3
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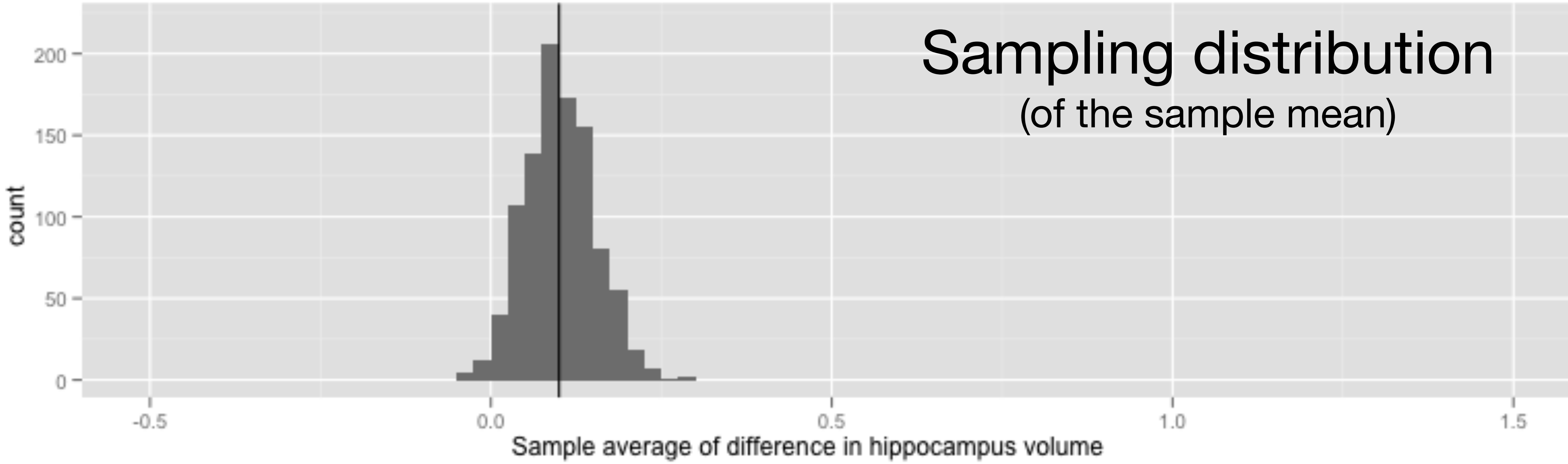
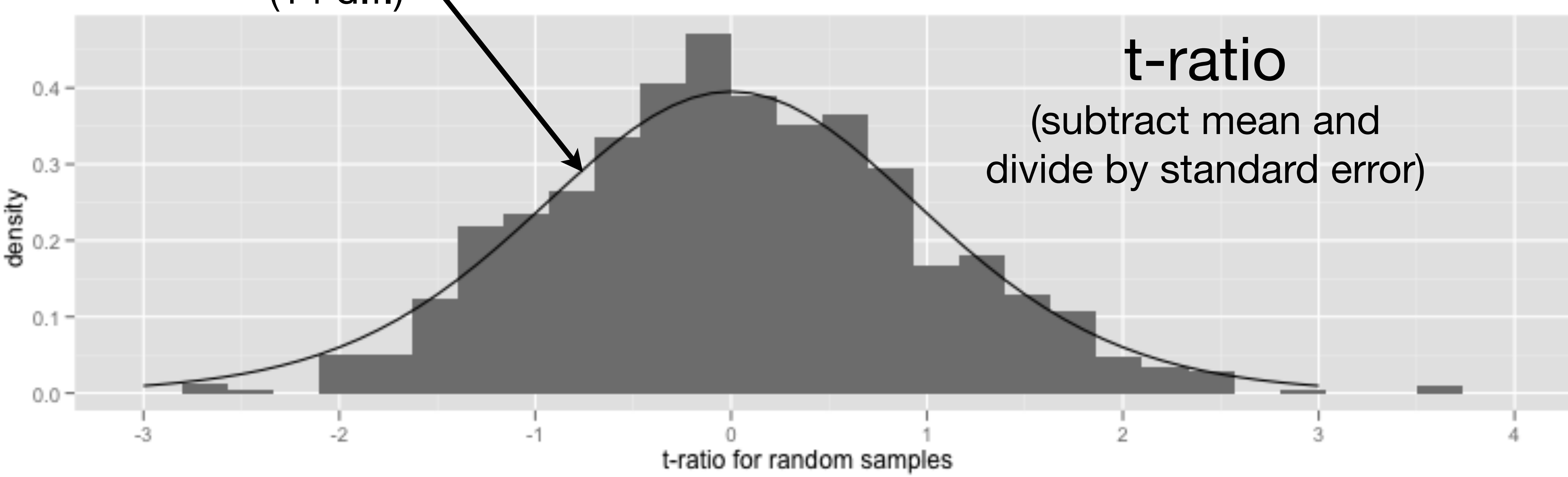
4
t-ratio for random samples



Student's *t*-distribution
(14 d.f.)



Is $\mu \neq 0$?

Remember the statistical justice system

Null hypothesis: The mean difference in hippocampus volume between schizophrenic and non-schizophrenic twins is zero. $\mu=0$

(describes what innocents look like)

Alternative hypothesis: The mean difference in hippocampus volume between schizophrenic and non-schizophrenic twins is not zero. $\mu\neq0$

(the crime)

Use the t-ratio as our test-statistic

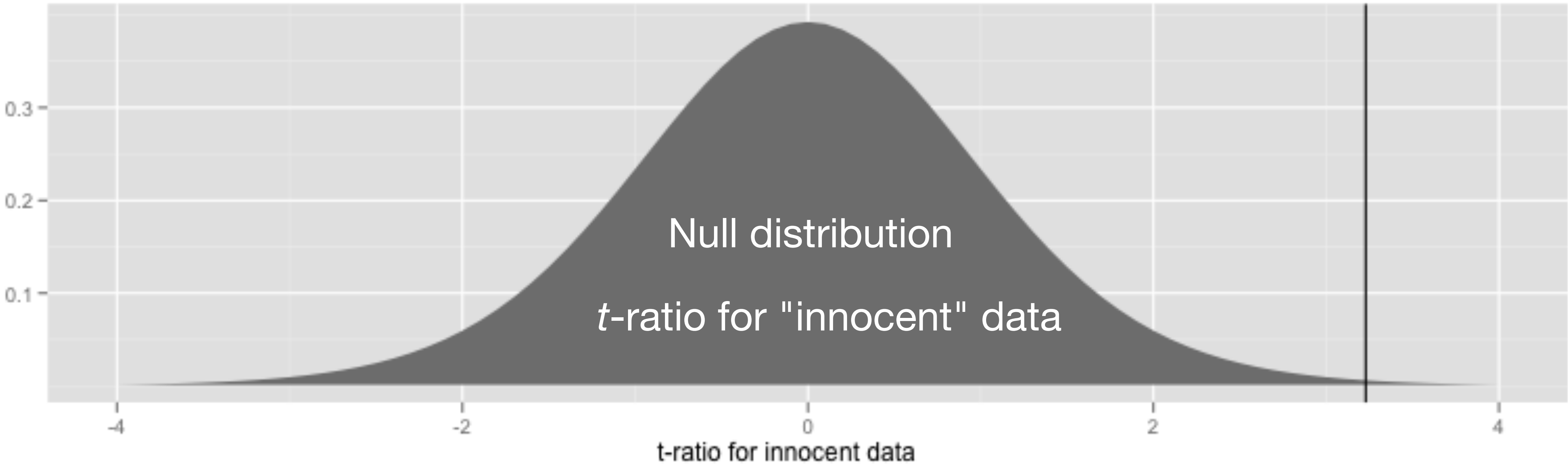
Innocents have μ=0, so the t-ratio $= \dfrac{\overline{Y}_{innocent} - 0}{\text{SE}}$

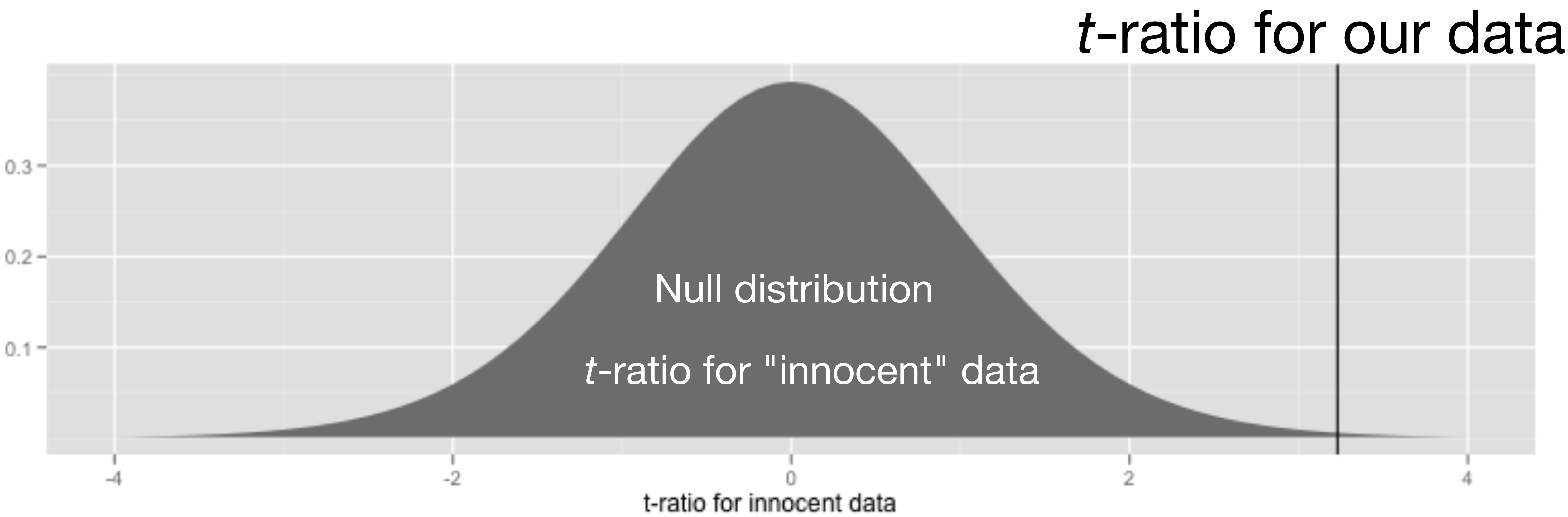
can be described by Student's t-distribution with 14 degrees of freedom.

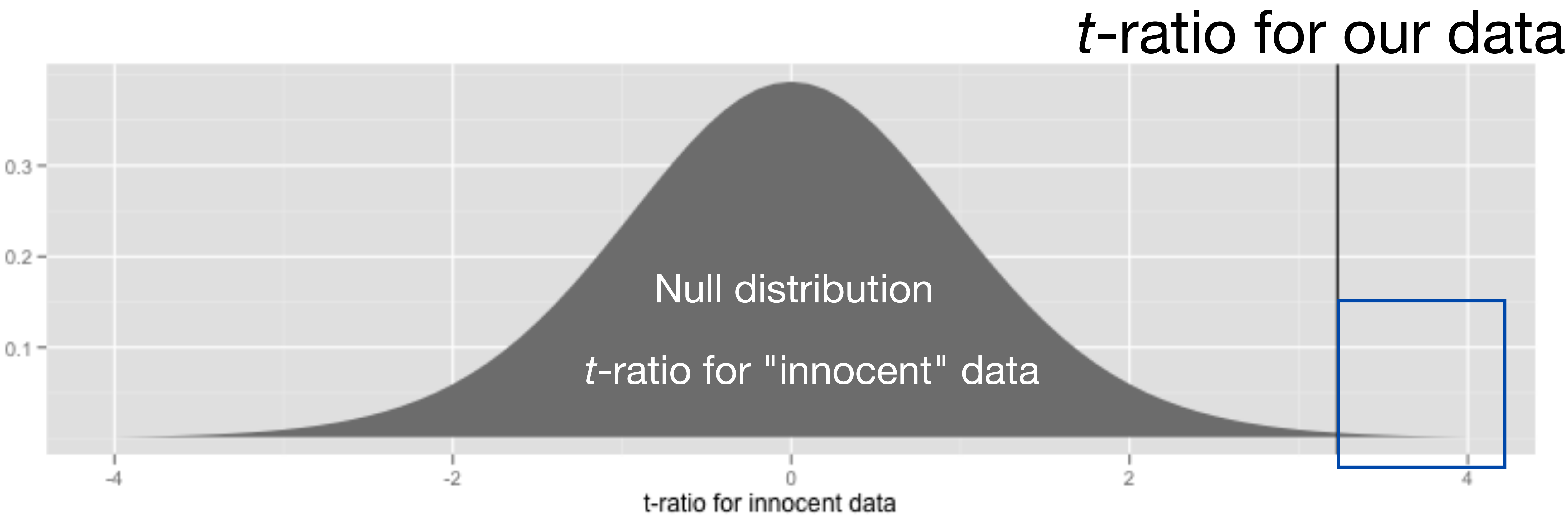
The evidence for our data is the same t-ratio for our data $\dfrac{\overline{Y} - 0}{\text{SE}} = 0.199/0.0615 = 3.236$

```
(mean(diffs) - 0 ) / (sd(diffs)/sqrt(length(diffs)))
```

When we are using the *t*-ratio as our test-statistic, we call it the *t*-statistic


0.3
0.2
0.1
Null distribution
t-ratio for "innocent" data
-4
-2
0
2
4
t-ratio for innocent data


t-ratio for our data
0.3
0.2
0.1
Null distribution
t-ratio for "innocent" data
-4
-2
0
2
4
t-ratio for innocent data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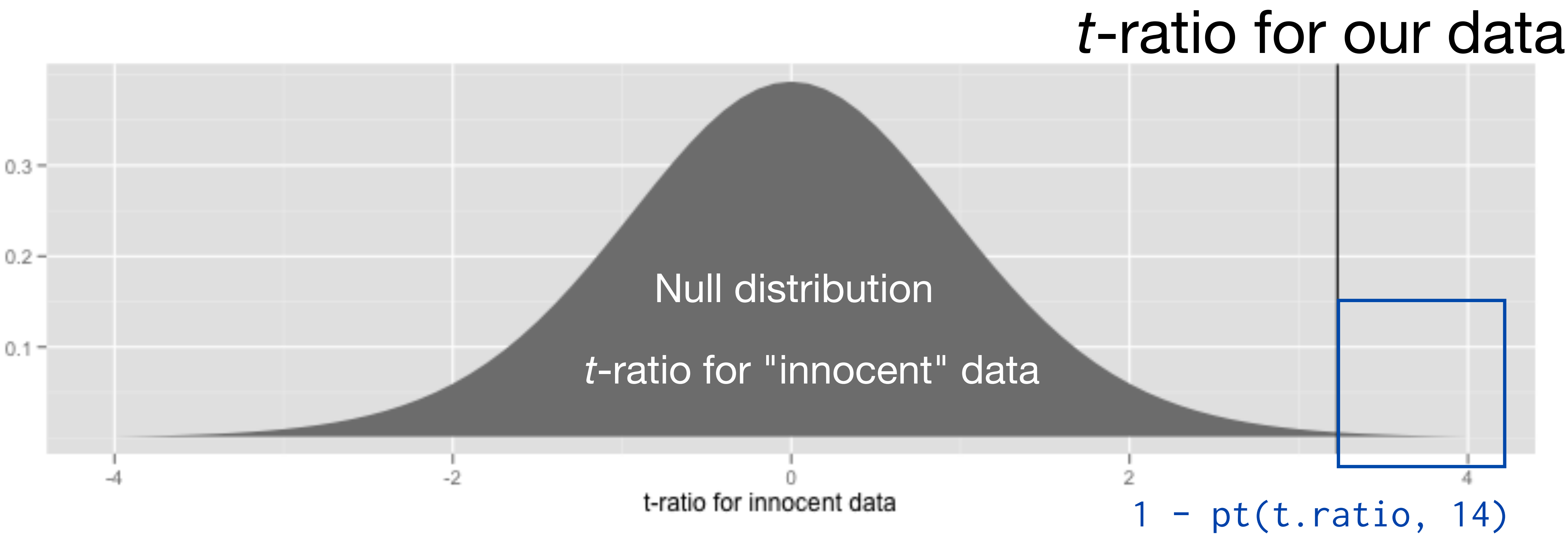

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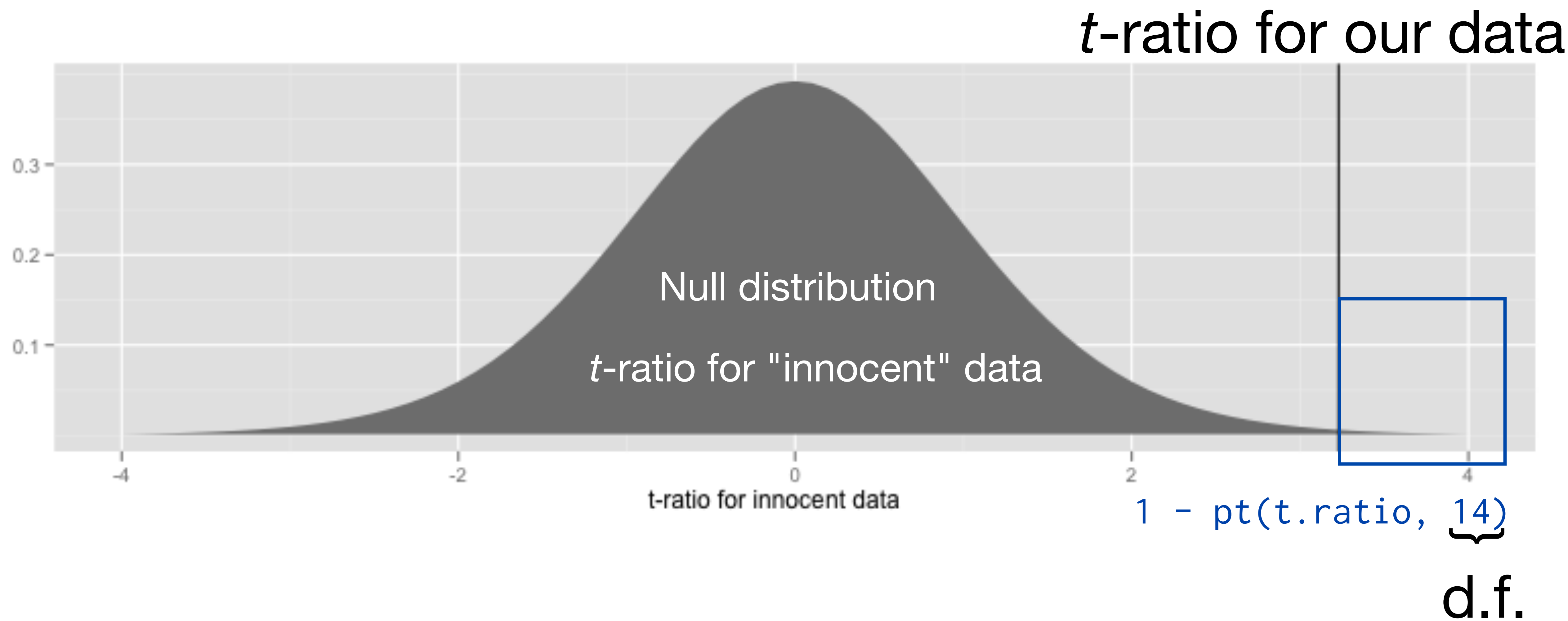
4
t-ratio for innocent data
1 - pt(t.ratio, 14)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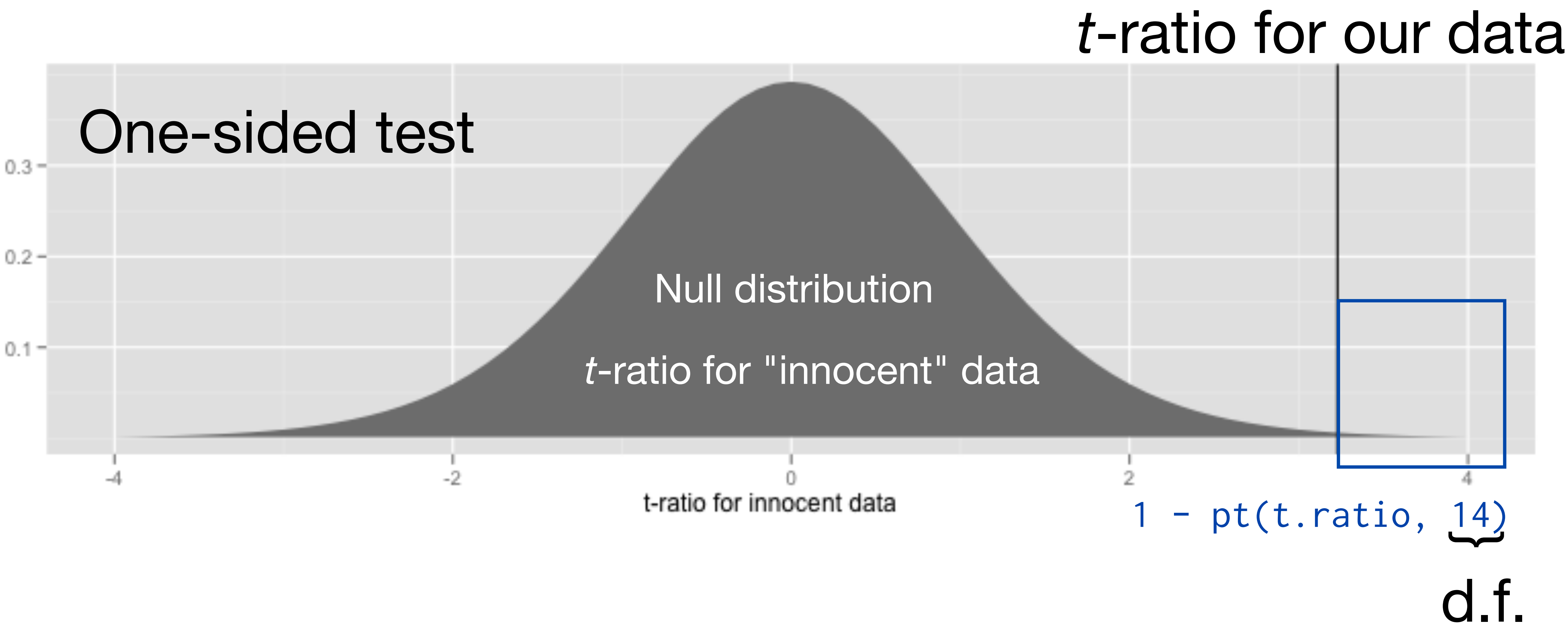
t-ratio for innocent data
1 - pt(t.ratio, 14)
d.f.


t-ratio for our data
One-sided test
0.3
0.2
0.1
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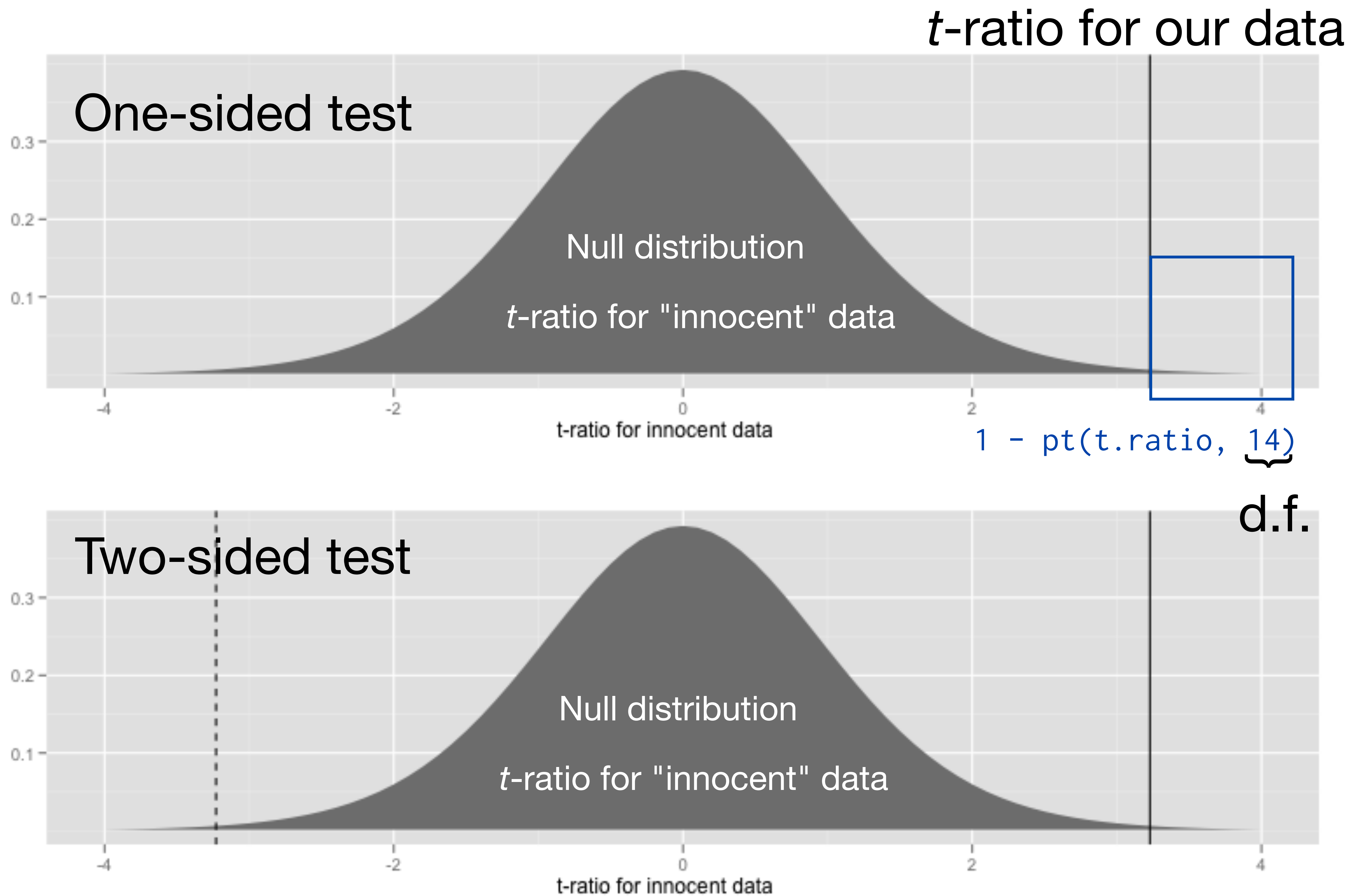
t-ratio for innocent data
1 - pt(t.ratio, 14)
d.f.


t-ratio for our data
One-sided test
Null distribution
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d.f.
Two-sided test
Null distribution
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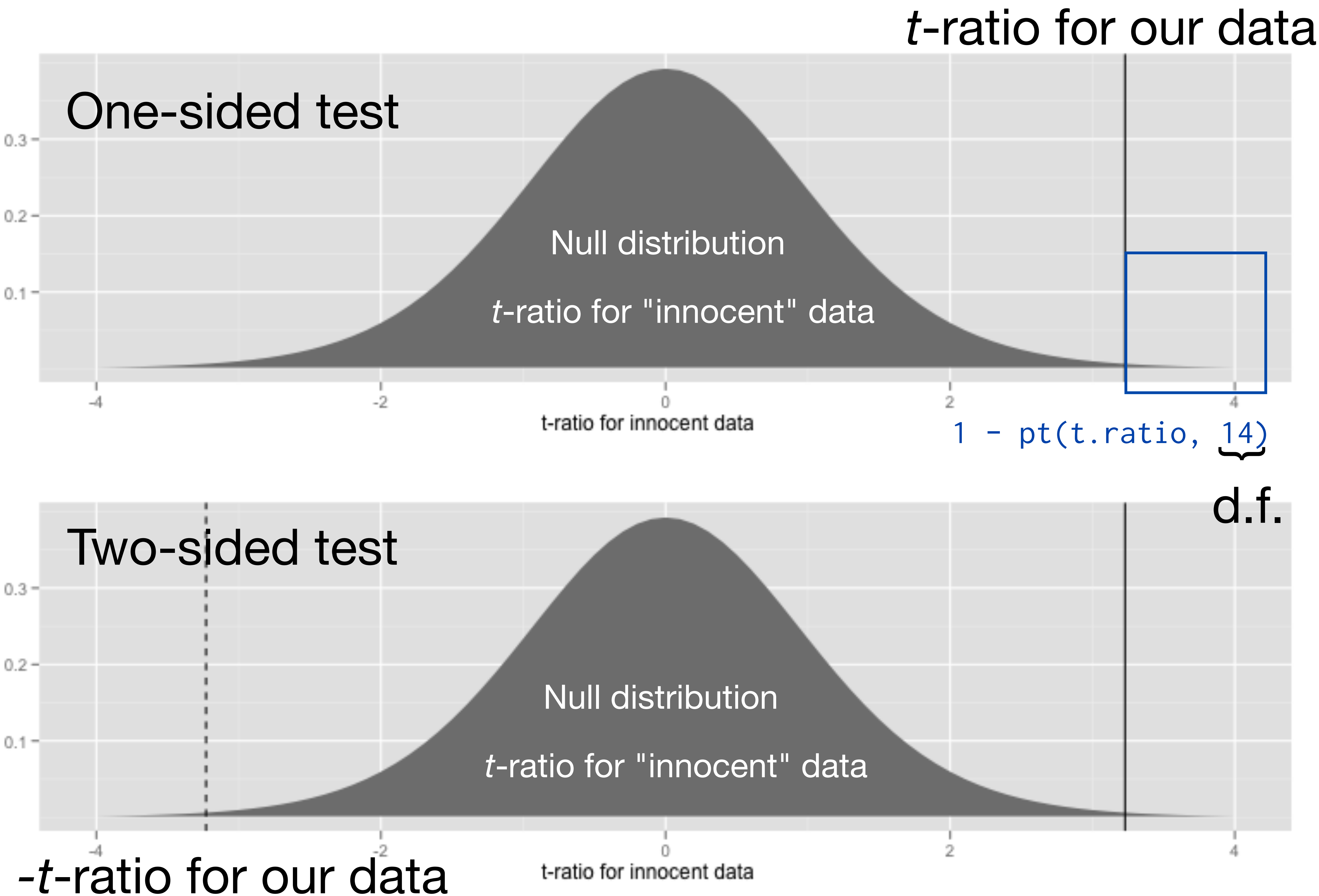
t-ratio for innocent data


t-ratio for our data
One-sided test
Null distribution
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Two-sided test
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t-ratio for "innocent" data
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0.1
-4
-2
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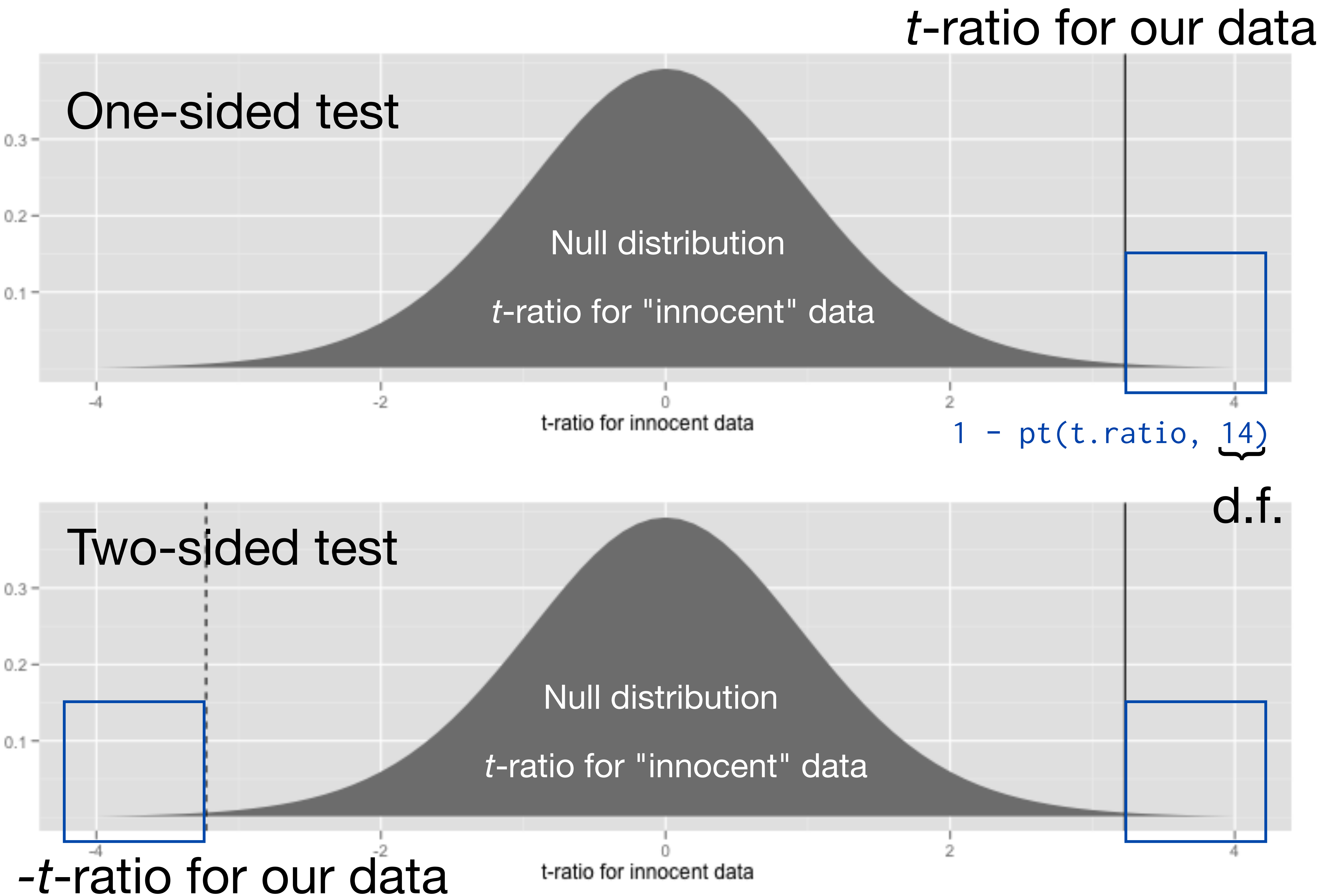
t-ratio for innocent data
-t-ratio for our data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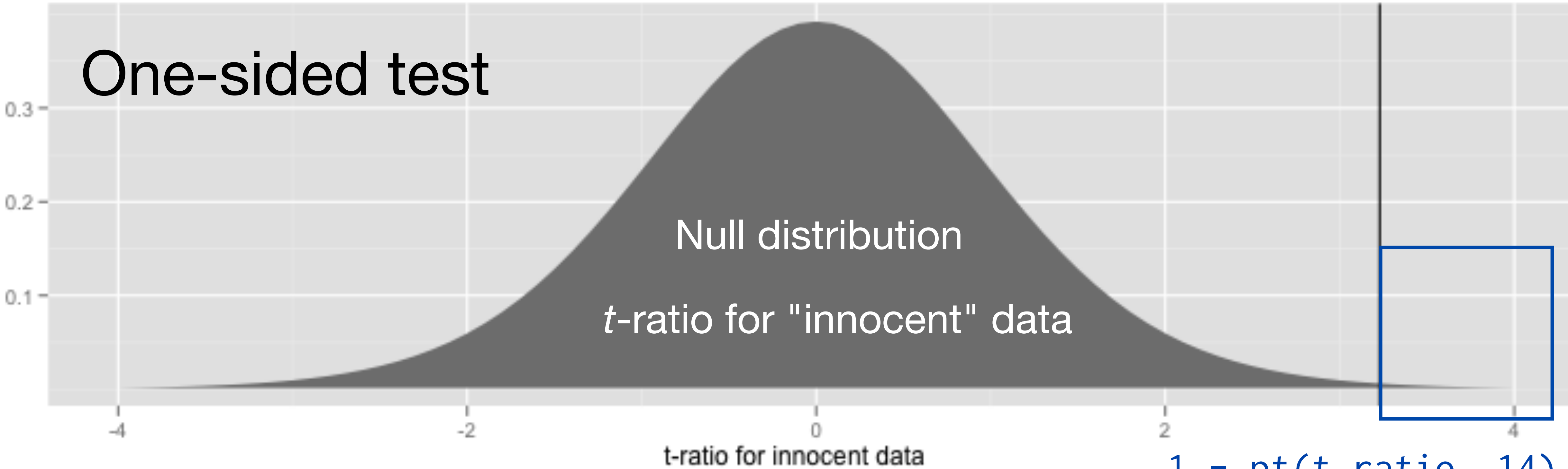
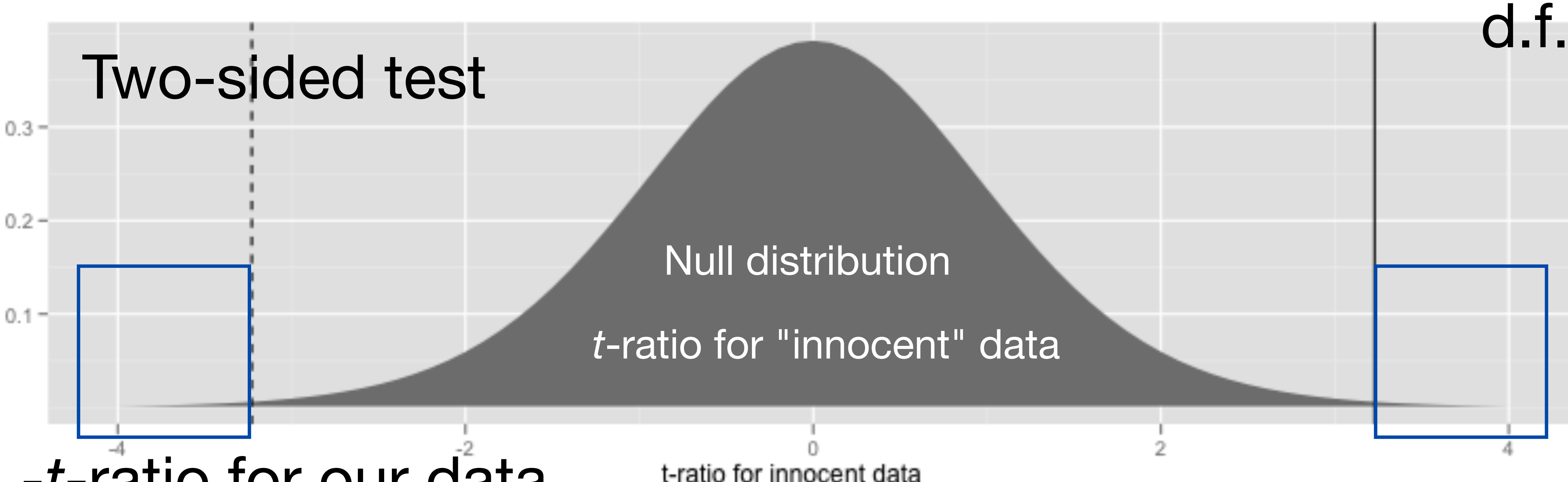



```
1 - pt(t.ratio, 14)
```

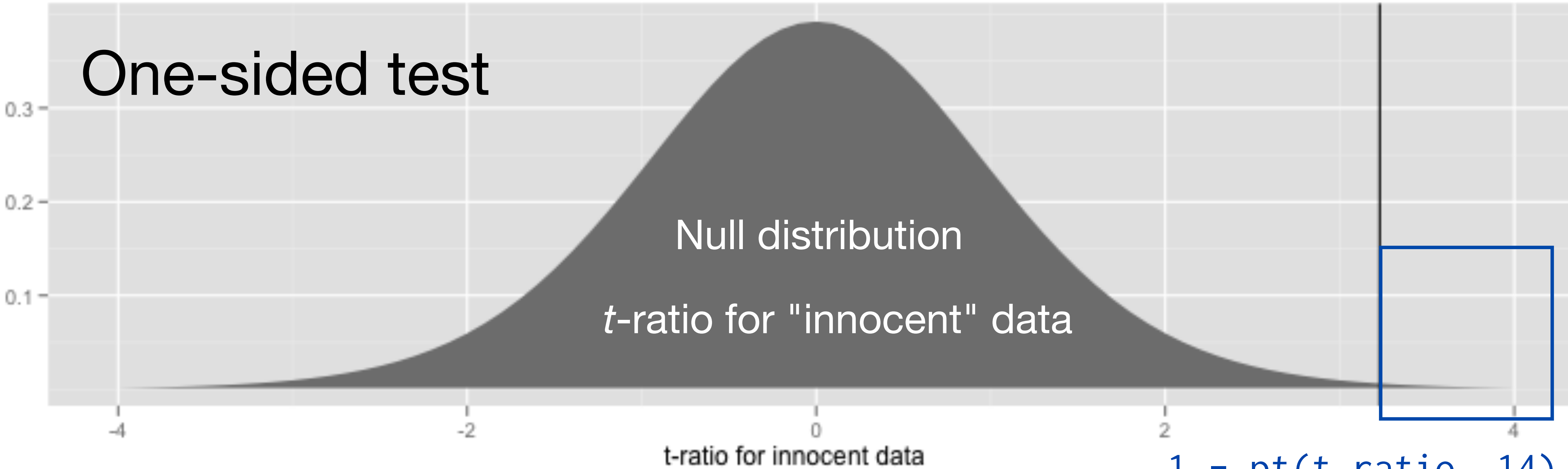
d.f.



-*t*-ratio for our data

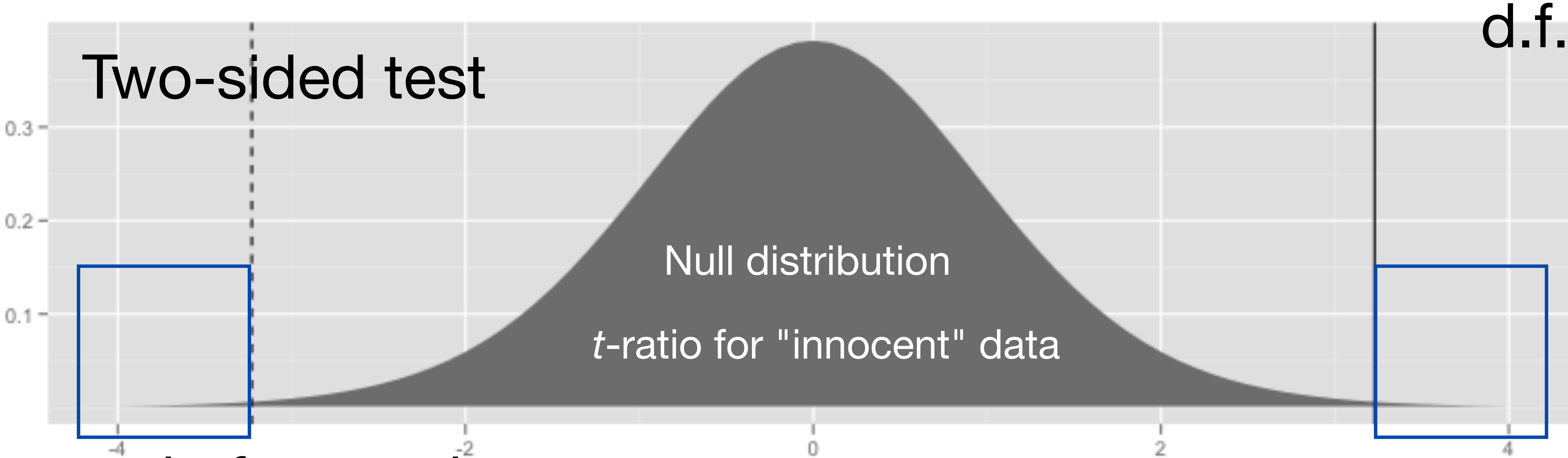
```
2 * (1 - pt(t.ratio, 14)
```

t-ratio for our data



`1 - pt(t.ratio, 14)`

d.f.



-*t*-ratio for our data

p-value = 0.006

`2 * (1 - pt(t.ratio, 14)`

What's a likely range for μ?

$$\overline{Y} \pm t_{n-1}(0.975) \times \mathrm{SE}_{\overline{Y}}$$

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Think about what hypotheses I could have tested for μ.

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Think about what hypotheses I could have tested for μ.

If I rejected the hypothesized μ, then it isn't a likely value, if I failed to reject then it is. Leads to an interval of likely values.

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If my significance level is 0.05, this leads to a 95% confidence level.

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$$\overline{Y} \pm \boxed{t_{n-1}(0.975)} \times \mathrm{SE}_{\overline{Y}}$$

What *t*-statistic has 97.5% of innocents below it?

$$\overline{Y} \pm t_{n-1}(0.975) \times \mathrm{SE}_{\overline{Y}}$$

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```
qt(0.975, 14)
```

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What *t*-statistic has 97.5% of innocents below it?

$$\texttt{qt(0.975, } \underbrace{\texttt{14}}_{\text{d.f.}}\texttt{)}$$

$$\overline{Y} \pm \boxed{t_{n-1}(0.975)} \times \text{SE}_{\overline{Y}}$$

What *t*-statistic has 97.5% of innocents below it?

```
qt(0.975, 14)
```

14 = d.f.

$$\begin{aligned} \overline{Y} \pm t_{n-1}(0.975) \times \text{SE} &= 0.199 \pm 2.145 \times 0.615 \\ &= (0.067, 0.331) \end{aligned}$$

```
mean(diffs) + qt(0.975, 14) * (sd(diffs)/sqrt(length(diffs)))
mean(diffs) - qt(0.975, 14) * (sd(diffs)/sqrt(length(diffs)))
```

$$\overline{Y} \pm \boxed{t_{n-1}(0.975)} \times \mathrm{SE}_{\overline{Y}}$$

What *t*-statistic has 97.5% of innocents below it?

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$\underbrace{14}$ d.f.

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```
mean(diffs) + qt(0.975, 14) * (sd(diffs)/sqrt(length(diffs)))
mean(diffs) - qt(0.975, 14) * (sd(diffs)/sqrt(length(diffs)))
```

We estimate the hippocampus in the schizophrenic twin is 0.2 cm^3 smaller than in the non schizophrenic twin. A 95% confidence interval for the difference is 0.07cm^3 to 0.33 cm^3

Interpretation of 95% CI

The true value of μ is either inside or outside this interval.

95% of the time we do this procedure our interval will contain the true value of μ.

t.test(diffs)

```
> t.test(diffs)

   One Sample t-test

data:  diffs
t = 3.2289, df = 14, p-value = 0.006062
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 0.06670409 0.33062922
sample estimates:
mean of x
0.1986667
```