Stat 411/511

WRAPPING UP T-TOOLS

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Today

Review hypotheses

Confidence intervals for two sample test

Assumptions of *t*-tools

Your turn

Which of the following are statistics and which are parameters:

- the sample average
- $\mu_2 - \mu_1$
- the standard deviation in humerus length of 24 sparrows that perished
- $\overline{Y}_2 - \overline{Y}_1$
- the median difference in brain volume for the 15 schizophrenic twins
- $\overline{Y}$
- the population treatment effect of the fish oil diet
- σ
- s
- the population mean

Statistical hypotheses are only made about **parameters**.

I have posted a flowchart to help guide you in writing hypotheses for two sample comparisons, and how to write the statistical summary.
Also see it for hints on the Fish Oil study.

In this class the **null** will always be exact, it contains an =.
The **null** should not contain a ≤, <, >, ≥.

Tip

Make **group 1** the group with the **lower** average.

Then:

- $\overline{Y}_2 - \overline{Y}_1$ will be positive
- you always calculate p-values using one or two times `(1 - pt(t.stat, df))`

  (as opposed to `pt(t.stat, df)`)

```
t.test(peri, surv, var.equal = TRUE)
```

Assume population standard deviations are the same

```
	Two Sample t-test

data:  peri and surv
t = -1.777, df = 57, p-value = 0.0809
alternative hypothesis: true difference in means
is not equal to 0
95 percent confidence interval:
 -21.446053   1.279386
sample estimates:
mean of x mean of y
 727.9167  738.0000
```

Confidence intervals for two-sample t-test

$$(\overline{Y_2} - \overline{Y_1}) \pm t_{n_1+n_2-2}(0.975) \times \mathrm{SE}_{\overline{Y_2}-\overline{Y_2}}$$

10.08 ± `qt(0.975, 24 + 35 - 2)` × 5.67

10.08 ± 2.002 × 5.67

10.08 ± 11.35

(-1.27, 21.43)

95% of values in a Student's t-distribution with 57 degrees of freedom, fall between -2.002 & 2.002

$$(\overline{Y_2} - \overline{Y_1}) \pm t_{n_1+n_2-2}(0.975) \times \mathrm{SE}_{\overline{Y_2}-\overline{Y_2}}$$

Multiplier for

80% CI	`qt(0.90, 24 + 35 - 2) = 1.297`
95% CI	`qt(0.975, 24 + 35 - 2) = 2.002`
99% CI	`qt(0.995, 24 + 35 - 2) = 2.666`
99.9% CI	`qt(0.9995, 24 + 35 - 2) = 3.470`

The **higher** the level of confidence the **wider** the interval

Confidence intervals and hypothesis tests

If an X% **confidence interval** for a parameter **does not contain zero**, then the **null hypothesis** that the parameter equals zero would be **rejected** at the 1-X% significance level.

If a **95%** confidence interval for the difference in population means does not contain zero, then the null hypothesis that the difference in population means is zero would be rejected at the **5%** significance level (the two-sided **p-value** for the test would be **less than 0.05**).

Your turn

A 95% confidence interval for the difference in the population mean head circumference between babies born to mothers who used marijuana and those who didn't is 0.61 to 1.19 cm (*didn't use* minus *used*).

TRUE or FALSE:

A hypothesis test for the null, that the mean difference is zero would be rejected at the 5% level.

The two-sided p-value would be less than 0.05.

The one-sided p-value would be greater than 0.025.

The null hypothesis is false.

The sample averages are different.

The population means are different.

Back to the creativity example

t-tests are motivated by **sampling from a population**, but also work well when the mechanism of chance is **random group allocation**.

Null: The mean treatment effect on the creativity score of being assigned the intrinsic motivation questionnaire compared to the extrinsic motivation questionnaire is zero.

t-statistic = $(\overline{Y}_2 - \overline{Y}_1 - 0) / SE_{\overline{Y}_2 - \overline{Y}_1} = 4.14 / 1.42 = 2.92$

`(1 - pt(2.92, 23 + 24 - 2))` = 0.0027 (one-sided p-value)

Assumptions of the *t*-tools

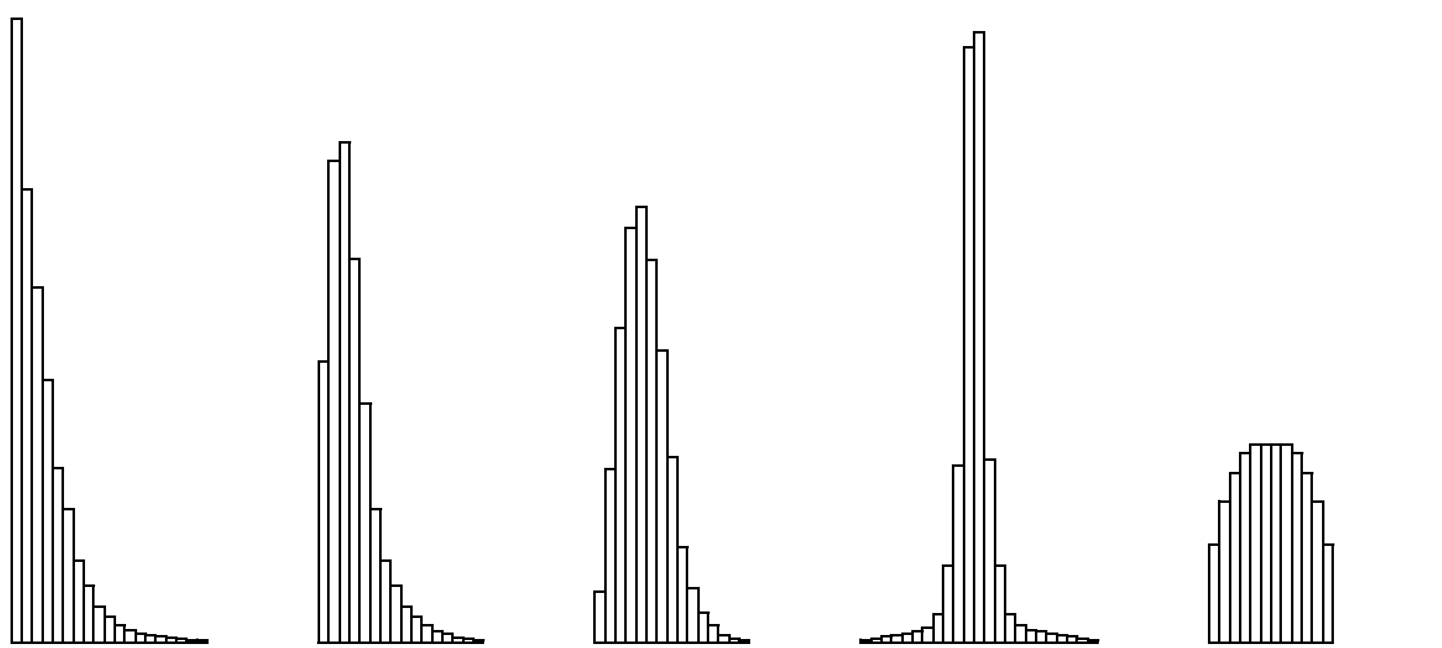
1. Normality of population
2. Equality of standard deviations
3. Independence of subjects

A test is **robust** to an assumption if the test is valid even if the assumption is not met.

Normality of population

Display 3.4 **p. 61**

Percentage of 95% confidence intervals that are successful when the two populations are non-normal (but same shape and SD, and equal sample sizes); each percentage is based on 1,000 computer simulations



n_1, n_2	strongly skewed	moderately skewed	mildly skewed	long-tailed	short-tailed
5	95.5	95.4	95.2	98.3	94.5
10	95.5	95.4	95.2	98.3	94.6
25	95.3	95.3	95.1	98.2	94.9
50	95.1	95.3	95.1	98.1	95.2
100	94.8	95.3	95.0	98.0	95.6

Normality of population

Check: by looking at histograms of samples.

Remedy:

use a transformation (Weds) **or**

use a non-parametric test (next week)