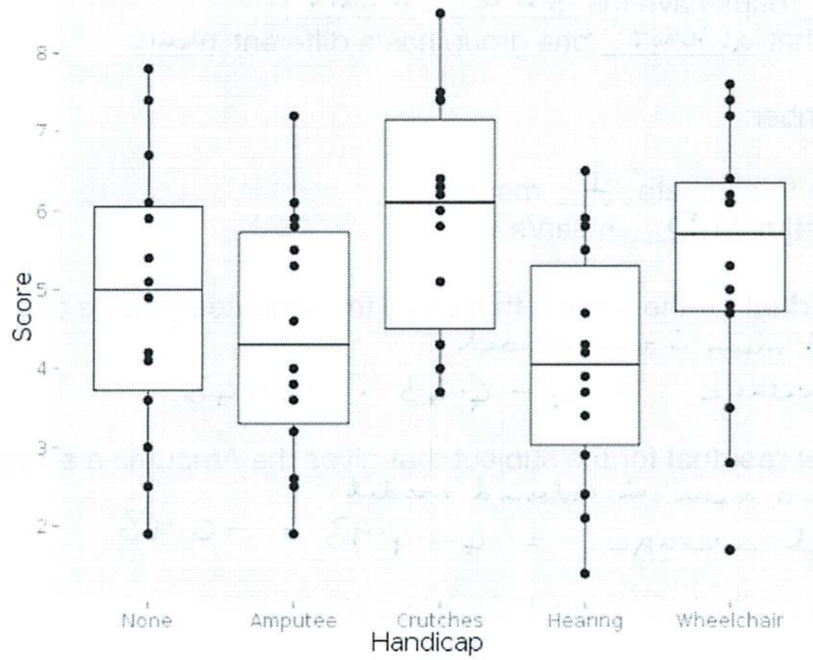
One way ANOVA worked example.

Discrimination Against the Physically Disabled Case Study 6.1 in Sleuth



Summary Statistics:

	None	Amputee	Crutches	Hearing	Wheelchair	Overall
Average	4.90	4.43	5.92	4.05	5.34	4.93
SD	1.79	1.59	1.48	1.53	1.75	1.72
Sample Size	14	14	14	14	14	70

We are going to perform a one-way analysis of variance to answer the question:
Do subjects evaluate qualifications differently according to the candidates handicap?

1. Read the study description in Sleuth 6.1.1. Are causal ~~casual~~ inferences possible? Are population inferences possible?

Since the subjects are randomly assigned to view the tapes, causal inferences are possible.

The subjects are undergraduates at a U.S. university, there is no mention of them being sampled randomly so population inferences are not possible.

2. Fill in the blanks:

The null hypothesis is that all groups have the same mean.
The alternative hypothesis is that at least one group has a different mean.

3. Fill in the blanks with a number:

In the reduced model we have to estimate 1 mean/s.
In the full model we have to estimate 5 mean/s.

4. What is the **full model residual** for the subject that gives the Amputee a score of 4.0?

4 - estimated mean in full model
= 4 - group average = 4 - 4.43 = -0.43

5. What is the **reduced model residual** for the subject that gives the Amputee a score of 4.0?

4 - estimated mean in reduced model
= 4 - overall average = 4 - 4.93 = -0.93

6. Calculate the pooled standard deviation, and it's degrees of freedom.

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2 + \ldots (n_I-1)s_I^2}{(n_1-1)+(n_2-1)+\ldots(n_I-1)}}$$

$$= \sqrt{\frac{13 \times 1.79^2 + 13 \times 1.59^2 + 13 \times 1.48^2 + 13 \times 1.53^2 + 13 \times 1.75^2}{13 + 13 + 13 + 13 + 13}}$$

$= 1.6325$

d.f. $= n - I$
$= 70 - 5$
$= 65$

6. Find the within group sum of squares and the total sum of squares using the shortcuts from homework 6.

Within group sum of squares $= s_p^2 \times$ (degrees of freedom for s_p)
$= 1.6325^2 \times 65$
$= 173.2287$

Total sum of squares = (Overall SD)^2 x (n - 1)
$= 1.72^2 \times (70-1)$
$= 204.1296$

8. Fill in the ANOVA table.

	Sum of squares	d.f	Mean Square	F-statistic	p-value
Between Groups	30.9009	4	7.7252	2.8987	0.028
Within Groups	173.2287	65	2.6651		
Total	204.1296	69			

If the null hypothesis is true, the F-statistic will be distributed like a F-distribution with 4 and 65 degrees of freedom.

The p-value can be found in R with: `1 - pf(` 2.8987, 4, 65 `)` = 0.0285

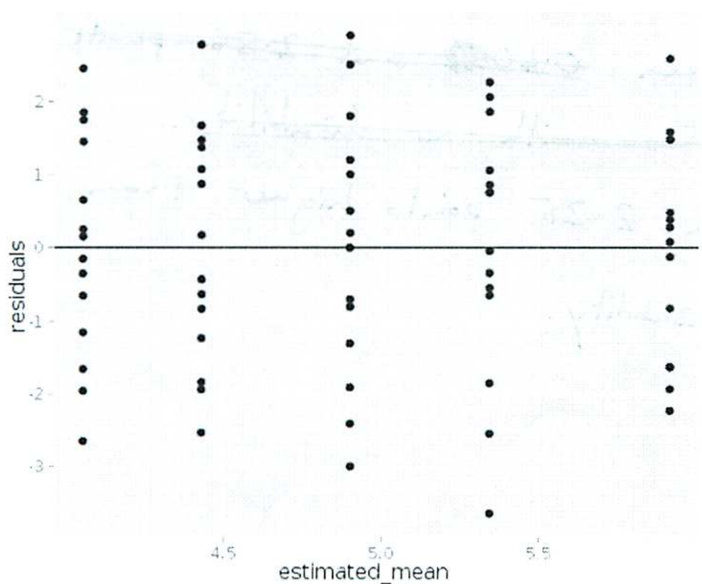
9. **Statistical summary**. Fill in the blanks:

There is moderate evidence that mean scores given to applicants with different disability status are not the same (F-test, p-value = 0.03).

10. What are the three assumptions of the one-way ANOVA?

1. Normality of populations
2. Equal SDs of populations
3. Independence of subjects within & between groups

Which can you check by examining the plot below?



1. Can check for gross violations of normality

2. Can check for evidence of unequal SDs.

Look OK!

11. A linear combination of the five means can be written as:

$Y = C_{none}\,\mu_{none} + C_{amputee}\,\mu_{amputee} + C_{crutches}\,\mu_{crutches} + C_{hearing}\,\mu_{hearing} + C_{wheelchair}\,\mu_{wheelchair}$

Each of the following comparisons can be written as a linear combination of means, write down the values of the constants (C's):

a. The difference in mean Score between a candidate with no disability and a candidate with crutches

$C_{none} = 1$, $C_{amputee} = 0$, $C_{crutches} = -1$, $C_{hearing} = 0$, $C_{wheelchair} = 0$

$\mu_{none} - \mu_{crutches}$

b. The difference in average of the mean Scores for candidates with no disability or a hearing disability and the average of the mean Scores for candidates with crutches or a wheelchair.

$C_{none} = \frac{1}{2}$, $C_{amputee} = 0$, $C_{crutches} = -\frac{1}{2}$, $C_{hearing} = \frac{1}{2}$, $C_{wheelchair} = -\frac{1}{2}$

$\frac{1}{2}(\mu_{none} + \mu_{hearing}) - \frac{1}{2}(\mu_{crutches} + \mu_{wheelchair})$

c. The difference in the mean Score for a candidate with no disability and the average of the mean Scores for candidates with crutches, a wheelchair or an amputee.

$C_{none} = 1$, $C_{amputee} = -\frac{1}{3}$, $C_{crutches} = -\frac{1}{3}$, $C_{hearing} = 0$, $C_{wheelchair} = -\frac{1}{3}$

$\mu_{none} - \frac{1}{3}(\mu_{crutches} + \mu_{wheelchair} + \mu_{amputee})$

d. Our estimate of Y for the linear combination in 11(c) is:

$g = C_{none}\bar{X}_{none} + C_{amputee}\bar{X}_{amputee} + C_{crutches}\bar{X}_{crutches} + C_{hearing}\bar{X}_{hearing} + C_{wheel}\bar{X}_{wheel}$

$= 1(4.9) + -\frac{1}{3}\times 4.43 - \frac{1}{3}\times 5.92 + 0\times 4.05 \cancel{+} \frac{1}{3}\times 5.34$

$= -0.33$

e. Construct a 95% confidence interval for Y for the linear combination in 11(c).

$$\mathrm{SE}_g = s_p\sqrt{\frac{C_1^2}{n_1} + \frac{C_2^2}{n_2} + \ldots + \frac{C_I^2}{n_I}}$$

$SE_g = 1.6325\sqrt{\frac{1^2}{14} + \frac{(-\frac{1}{3})^2}{14} + \frac{(-\frac{1}{3})^2}{14} + \frac{0^2}{14} + \frac{(-\frac{1}{3})^2}{14}}$

$\cancel{= 1.885}$ $= 0.504$

95% CI: $g \pm 2 \times \cancel{}\, 0.504$ — $qt(0.975, n-I)$

$= -0.33 \pm 2 \times \cancel{}\, 0.504$

$= (-1.338, 0.678)$

(Hint: qt(0.975, 65) = 2.00)

With 95% confidence, the mean score of a candidate with no disability is between 1.33 points smaller and 0.678 points larger than the average of the mean scores for candidates with crutches, wheelchair or an amputee.