# Stat 411/511

## Extra SS F-test

Nov 6 2015

Charlotte Wickham

stat511.cwick.co.nz

# Quiz #3

- Today noon - Monday noon
- Same format, two sections, 30mins each
- Short answer you need to be able to do exp(x)

# The ANOVA table

## a convenient way to lay the calculations out

**Display 5.10** **p. 127**

**Analysis of variance table: a test for equal mean percents of women in venires of seven judges; Spock data**

| Source of Variation | | Sum of Squares | df | Mean Square | F-Statistic | p-value |
|---|---|---|---|---|---|---|
| Between Groups | | | | | | |
| Within Groups | Full model | 1,864.45 | 39 | | | |
| Total | Reduced model | 3,791.53 | 45 | | | |

1. *Sum of squared residuals from fitting the full (separate-means) model*

2. *Sum of squared residuals from fitting the reduced (equal-means) model*

3. *degrees of freedom*: *n-I*, *n-1*

4. *Subtract the "Within" from the "Total "*

5. *A mean square is the ratio of a sum-of-squares to its degrees of freedom*

6. *The F-statistic is the ratio of the Between MS to the Within MS*

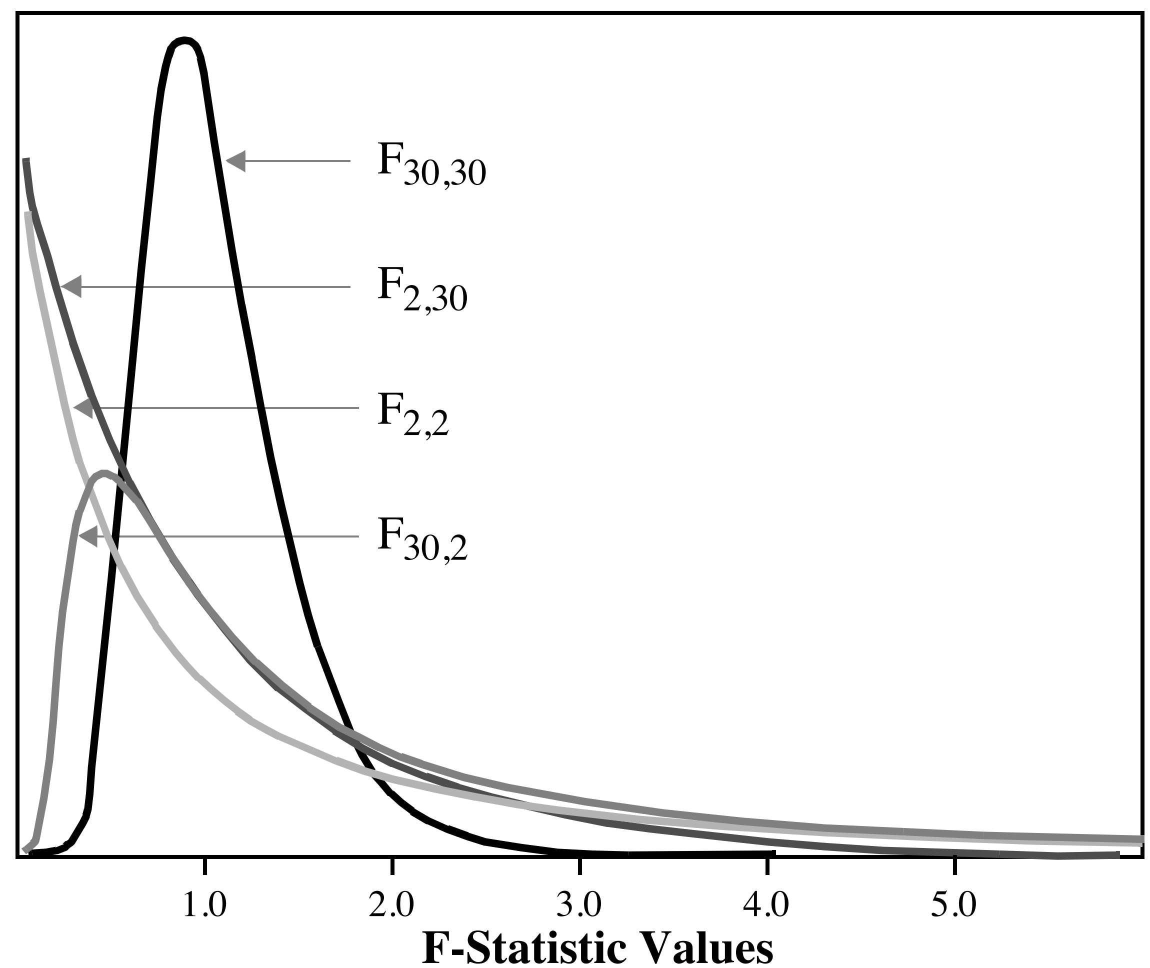
7. *The p-value comes from an F-distribution with and df*

***NOTE**: This is $s_p^2$*

# The p-value is always one-sided for the F-test

**Display 5.9** **p. 126**

**Four F-distributions, having different degrees of freedom**




Large F stats give evidence against the null hypothesis.

We have convincing evidence that at least one judge has a different mean percentage of women on their venires (one-way Anova F test on 6 and 39 degrees of freedom, p-value = 0.00006).

# One way anova in R

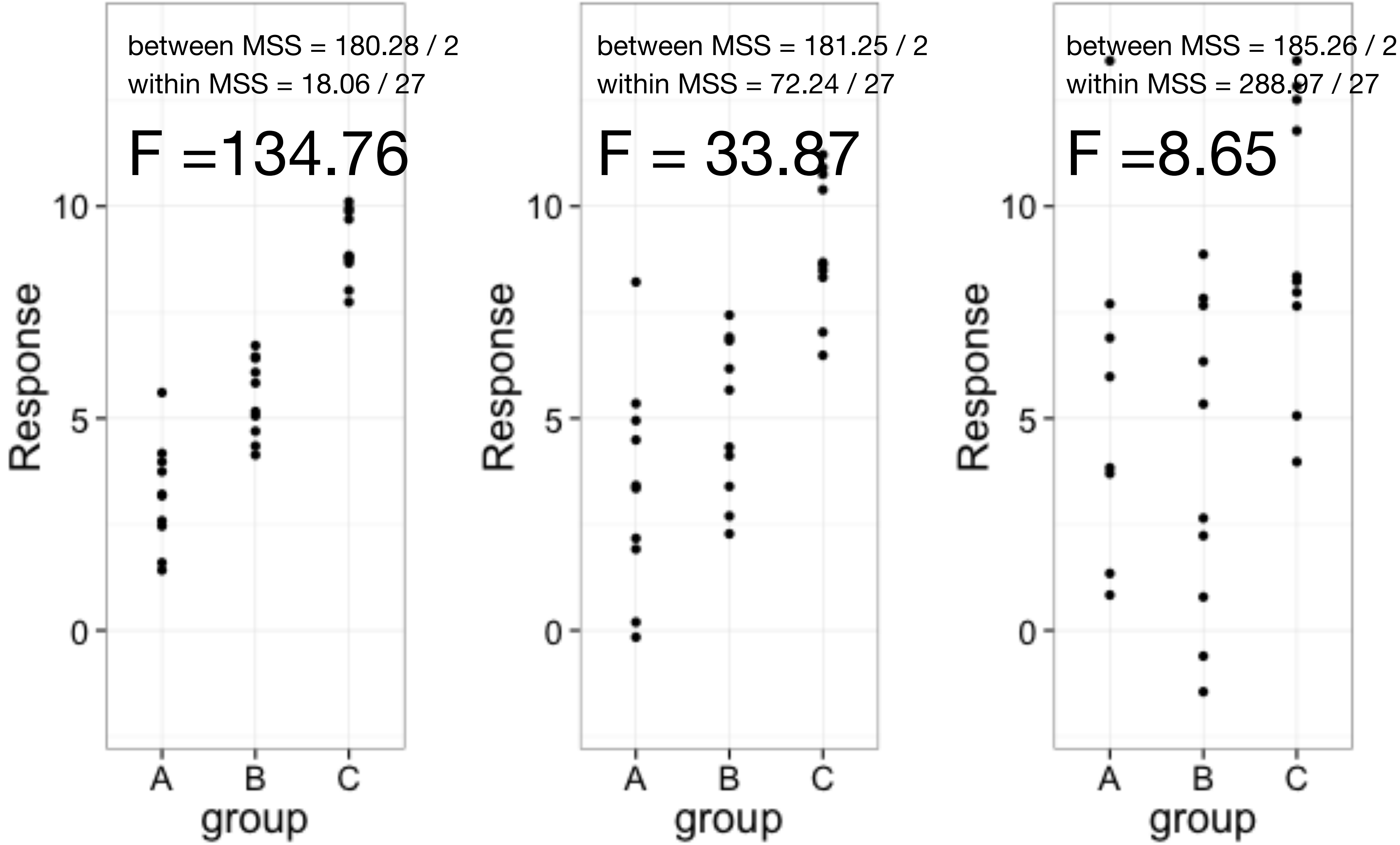
```
anova(lm(Percent ~ Judge, data = case0502))
```

response groups

```
Analysis of Variance Table

Response: Percent
          Df Sum Sq Mean Sq F value    Pr(>F)
Judge      6 1927.1  321.18  6.7184 6.096e-05 ***
Residuals 39 1864.5   47.81
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

This will test the separate means model against the equal means model




$p < 1x10^{-14}$ $p < 1x10^{-7}$ $p = 0.0012$

Which would you say gives more evidence of the groups coming from populations with different means?

# Spock's trial

There is evidence that at least one judge has a different mean percent of women. a one-way anova conclusion

But what we really want to argue is that Spock's judge has a different mean percent of women compared to all the other judges.

We argue this in two steps:

**Argument #1.** There's no evidence the other judges (A-F) have different mean percentage of women from each other. That means it makes sense to imagine they are all drawing venires at random from a population with mean percentage of women, $\mu_{0.}$

**Argument #2.** There is evidence that Spock's judge has mean percentage of women that is not the same as the other judges, i.e. it is not, $\mu_0$.

These aren't one-way anova conclusions, we still use the extra SS F-test to reach them, but with different full and reduced models.

An extra Sums of Squares F-test compares **two models**

**Full model**: a model that fully describes the set of alternatives.

fit and find RSS and df

**Restricted model**: a restriction of the full model imposed by the null hypothesis.

fit and find RSS and df

# Extra SS F-statistic

$$F = \frac{\text{Extra Sum of Squares}/\text{Extra degrees of freedom}}{\hat{\sigma}^2_{\text{full}}}$$

squared pooled sd from full model

find it from SS/df for full model

**Extra Sum of Squares** =

RSS from reduced model **-** RSS from full model =

reduction in residual sum of squares

**Extra degrees of freedom** =

df in reduced model **-** df in full model =

reduction in degrees of freedom

or how many **more** parameters does the full model have?

Under the **null** hypothesis (reduced model is true) the F-statistic has an F-distribution with $v_1$ and $v_2$ degrees of freedom.

v1 = extra df, v2 = df of full model

# Argument #1

**Null**: the population mean percentage of women is the same for all the **other** judges.

$\mu_{Spock} = \mu_1$, $\mu_A = \mu_B = \mu_C = \mu_D = \mu_E = \mu_F = \mu_0$

**Alternative:** at least one “other” judge has a different mean.

**Full model:** separate means for all judges.

**Null model:** two means, one for Spock, one for everyone else.

```
> # create new Spock or Not variable
> case0502$two_groups <- ifelse(case0502$Judge == "Spock's", "Spock's", "Other")

> # find two group averages
> case0502$two_group_average <- with(case0502, ave(Percent, two_groups))

> # sum of squared residuals
> with(case0502, sum((Percent - two_group_average)^2))
[1] 2190.903
```

**Argument #1** **Null**: the population mean percentage of women is the same for all the **other** judges.

# ANOVA Table

| | | Sum of squared residuals | d.f. |
|---|---|---|---|
| **Full model** | separate means model | 1864.46 | |
| **Reduced model** | two means model | 2190.9 | |

n = 46

n - # parameters

**Argument #1** **Null**: the population mean percentage of women is the same for all the **other** judges.

# Your turn: Find the F-statistic

| | | Sum of squared residuals | d.f. | MSS | F | p-value |
|---|---|---|---|---|---|---|
| Extra | | C: subtract A from B | F: subtract D from E | G: divide C by F | I: divide G by H | use R: `1- pf(I, F, D)` |
| **Full model** | separate means model | A 1864.46 | D 39 | H: divide A by D | | |
| **Reduced model** | two means model | B 2190.9 | E 44 | | | |

# Argument #1

We have evidence that any of the **other** six judges have a different mean percentage of women on their venires (extra sum of squares F-test on and degrees of freedom, p-value = ).

# Argument #2

**Null**: the population mean percentage of women for Spock is the same as the mean for all the other judges.

$$\mu_1 = \mu_0$$

**Reduced model:** there is **one** parameter, the mean for everyone.

equal means model

**Full model:** there are **two** parameters, the mean for Spock's judge, the mean of all the other judges.

two means model

**Argument #2** the population mean percentage of women for Spock is the same as the mean for all the other judges

# ANOVA Table

| | | Sum of squared residuals | d.f. |
|---|---|---|---|
| **Full model** | two means model | 2190.9 | 44 |
| **Reduced model** | equal means model | 3791.53 | 45 |

**Argument #2** the population mean percentage of women for Spock is the same as the mean for all the other judges

| | | Sum of squared residuals | d.f. | MSS | F | p-value |
|---|---|---|---|---|---|---|
| | what's left? | C: subtract A from B | F: subtract D from E | G: divide C by F | I: divide G by H | <0.0001 |
| **Full model** | two means model | 2190.9 | 44 | H: divide A by D | | |
| **Reduced model** | equal means model | 3791.53 | 45 | | | |

# Argument #2

We have evidence that Spock's judge has a different mean percentage of women on his venires than all the other judges (extra sum of squares F-test on and degrees of freedom, p-value = ).

Optional

# Anova in R

To test the full model against another model

```
> # full model
> m1 <-  lm(Percent ~ Judge, data = case0502)

> # all other judges equal
> m2 <-  lm(Percent ~ two_groups, data = case0502)

> # gives the anova table for two means versus full model
> anova(m1, m2)

Analysis of Variance Table

Model 1: Percent ~ Judge
Model 2: Percent ~ two_groups
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1     39 1864.5
2     44 2190.9 -5   -326.46 1.3658 0.2582
```

A variable I made